DEVELOPMENT OF A SIMPLIFIED ENGINEERING DESIGN EQUATION FOR A THERMOELECTRIC HEATING/COOLING DEVICE

by

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B. S., The Pennsylvania State University, 1941

A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Agricultural Engineering

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Manhattan, Kansas

1967

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INTRODUCTION

Utilization of electricity on the farm and in the farm home is increasing along with the development of new equipment as a result of further application of basic principles which were discovered 50 to 100 or more years ago. One such electro-physical principle, known as thermoelectricity, has created interest in recent years with regard to the use of the Peltier effect for cooling and/or heating purposes. This is the reverse process to thermoelectric power generation (Seebeck potential) of direct current by heating alternate junctions of pairs of dissimilar metals or semi-conductors, the latter being required for high efficiency. Utilization of the refrigerating affect achieved in this manner has been limited to laboratory experiments due to the minuscule nature of the Peltier effect.

A French physicist, Jean C. A. Peltier in 1834 first observed the reverse of the thermoelectric phenomenon discovered in 1832 by Thomas J. Seebeck, a German physicist. At that time it would have been theoretically possible to generate electricity with thermocouples at an efficiency of about 3 percent, which would have been as good as the best steam engines available.

During the past decade or so progress in the development of thermoelectric materials have produced more efficient semi-conductor compounds. Lead Telluride is one such compound that is now being used in such applications as water coolers, ice cube makers, air conditioners, and numerous instrumentation devices. The material development has reached a plateau during the past several years. However the method of assembling these materials into thermoelectric junctions has continued to advance and thus further improve the efficiency of the devices.

There is also continued advancement in mass production techniques for assembling various numbers of thermoelectric junctions into a module. Such techniques will further reduce the cost per Btu of cooling capacity which will make larger thermoelectric devices more competitive with conventional refrigeration systems.

The reverse cycle refrigeration system or "heat pump" as it is commonly called, is capable of transferring heat from a low energy level to a high energy level. This device can transfer more heat (Btu/Hr) than the heat equivalent (Btu/hr) of the system power input. The ratio of the heat transferred to the heat equivalent of the power input is called the coefficient of performance or C.O.P. of the device. A thermoelectric "heat pump" invariably has a heating coefficient of performance greater than unity. At the present state of thermoelectric material development, the cooling coefficient of performance is usually less than unity. However it is predicted that as more efficient thermoelectric materials are developed and better design methods are found for applying them in heat transfer devices, a cooling coefficient of performance greater than unity can be accomplished.

Also this transfer of heat can be accomplished by thermoelectric devices which have no moving parts. The refrigeration effect and heat transfer is accomplished solely by the flow of electrons through the thermoelectric materials. Today's agricultural scientists apply new principles and methods of heat transfer to their design needs. The improvement of conventional systems by eliminating moving parts must be considered to be of major importance. Several theoretically possible thermoelectric applications in agriculture are cooling milk and heating water (or air); milk cooling and pasteurization; humidity and dew point controls; equipment operator air conditioning; animal body cooling or heating; dehumidifiers;

electronic control element cooling, water cooling; refrigeration of semen, antibiotics and vaccines and small isolated power supplies.

THE PROBLEM

The design of thermoelectric heating/cooling systems has heretofore been investigated for systems that generally have somewhat fixed conditions on both sides of the heat transfer device. Such systems are represented by water coolers, refrigerators, air conditioners, ice cube machines and freezers.

Methods of estimating the cooling capacity, power requirements and other factors have utilized complicated charts and computer programs. There appears to be very little information on engineering design equations for determining the relationship of the important variables in a transient system. Such a device can be represented by a milk cooler/water heater or a milk pasteurizer/water cooler, milk cooler/water heater combination.

It would appear that a general engineering design equation which relates the important variables would be a useful tool for the engineer.

OBJECTIVES

The objectives of the study were:

1. To investigate the development of a general prediction equation that describes the relationship of the important variables in a thermoelectric heating/cooling system.
2. To verify the general prediction equation by laboratory experiments.

The experimental research was planned to incorporate two flat-plate liquid heat exchangers with a 12-couple thermoelectric module mounted between these heat exchangers. The thermoelectric module is of a type that is commercially available at this time.

REVIEW OF LITERATURE

Only one reference has been found regarding the use of thermoelectricity in agriculture. Golubyatnikov (10) reports on a thermoelectric milk cooler/water heater unit being tested in Russia. He states,

> Its relevant data are: throughput 100-150 lit./hr. (26-40 gal./hr.) (or approximately 220-340 lbs./hr.) with milk storage capacity up to 2000 Kg (510 gal.); refrigerating capacity, 2700-3000 Kcal./hr. (10,700-11,900 Btu./hr.); cooling coefficient E (C.O.P.) = 1.36 - 1.92; power requirement = 0.9-2.7 KW, d.c. current I = 140 - 150 A (amperes); milk can be cooled to a temperature of 8-5°C (46-41°F); the energetic characteristic of the semiconductor material Z = 1.70 x $10^{-3}/^{\circ}$C; the weight of semiconductor material used is 6.0-7.5 Kg. (13-16.5 lbs.); the number of thermocouples: up to 1000.
>
> The thermoelectric refrigerator constitutes a thermoelectric battery of circular high-current thermocouples consisting of a ternary alloy Te Bi Se negative branch and a Te Bi Sb positive branch, connected in series. It was found that the inertia of circular thermocouples is small, and that they can provide a high power output with stable characteristic.
>
> By this circular thermocouple design, resulting in much simplified manufacturing technology, semiconductor power systems can be realized . . .

Golubyatnikov further describes the thermoelectric rectifier power supply water cooler. Thermostats control the milk input and output for the device. The milk is first pasteurized and then cooled. The heat transfer mechanism used provides a solution of the problem of the cooling of milk with material having a small figure of merit.

He states further,

> The counter-convective heat exchange is effected (provided) by the appropriate direction of the milk and cooling water flows,

> and by passing the liquid to be cooled over the radiator surface of the generator (thermoelectric cooler). This assures maximum heat loss and a maximum heat transfer coefficient and consequently an increase in generator efficiency. Thus an energetic characteristic $Z = 1.70 \times 10^{-3}/^{o}C$ obtained by the USSR Semiconductor Institute, is perfectly adequate for solving the thermal-engineering and hydromechanical problems connected with the design of an economically operating thermoelectric refrigerator-heater for dairy farms.

A paper prepared by Charles L. Feldman et al. (7) explains a method by which cascaded peltier coolers can be optimized under certain specified conditions such as hot junction temperature. The figure of merit was assumed to be $2.8 \times 10^{-3}\ K^{-1}$ and T_h = 300K. A low temperature freezer was designed from theoretical calculations using bismuth telluride thermoelectric modules.

A report (1) by Dr. E. F. Cox of the Whirlpool Corporation states, "Except for economics, thermoelectric heat pumping should be excellently suited to distillation processes." He presents an abbreviated and simplified theoretical analysis to determine the optimum thermoelectric material properties to maximize efficiency of salt water still operation. Several ideas are presented for making short thermoelements and junctioning them to bridges. The possibility of thin films of metals or semiconductors are discussed and studies along these lines are recommended as a wide-open area for experimental research.

A series of reports by the United States Naval Research Laboratory Washington, D.C. (2) review the status of thermoelectric materials and device developments. The final report (NRL Memorandum Report 1404) in this series discusses briefly Peltier heat pumps, parameters and device possibilities. The report states, "Materials developments now make possible the construction of practical reversible Peltier heat pumps and simple refrigerators for a variety of military applications."

D. C. Siegla et al. (19) have analyzed the heat transfer characteristics of a thermoelectric module for refrigeration. A one dimensional and a three dimensional analysis are described. In both analyses the relationship of junction temperature and dimensions of the thermoelectric element are evaluated with respect to the heat transfer capability of the module.

In several progress reports by the Carrier Corporation to the Department of the Navy (22), research is described relating to design of submarine air conditioning units. An analog computer study is described which program produces operation data based on selected values of junction temperature differences. Thermoelectric submarine air cooling systems utilizing sea water exchangers are estimated to be 71 percent as heavy as a conventional reciprocating refrigeration system.

A NRL status report by Dr. J. W. Davisson and Joseph Pasternak (3) describes theoretical analysis by analog computer programs of thermoelectric devices for use in submarines. In each design certain factors were held constant such as sink water temperature, air flow through cooling coil or cooling capacity. Therefore all the important variables are not combined into one solution.

Another NRL status report by Dr. J. W. Davisson and Joseph Pasternak (4) summarizes thermoelectric materials characteristics and includes such parameters as figure of merit, Seebeck coefficient, electrical resistivity, thermal conductivity and others.

A report by E. W. Frantti, Westinghouse Electric Corporation (8) describes the analysis of a thermoelectric air conditioning system for submarines. Test loops were set up to measure the operating characteristics of a thermoelectric module in the laboratory. Various relationships were derived from laboratory measurements. However the relationships were not of a mathematical form such that the variables could be combined. For certain fixed conditions, curves are presented which, for example, show the relationship of heat pumping rate and C.O.P., to current, chill water flow or heat sink water flow.

The transient performance of a thermoelectric couple under step-current control is described in a paper by W. F. Stoecker and J. B. Chaddock (20). The temperature of the cold junction with respect to time is shown following a step-change in current. For example this characteristic is shown for step-change from 0 to 10 amperes and from 10 to 20 amperes and vice versa. In all cases the hot junction temperature was held constant at 310^{o}K.

Magazine articles by Alwin B. Newton (15) discuss the selection of materials and prediction of performance for thermoelectric systems. Conventional charts are presented which show performance for a fixed sink temperature. Analysis of other heat transfer variables is not included. The author points out a recent development of ceramic interfaces between the module junctions and the surface being cooled or heated. This development has reduced the temperature drop across such interfaces from about 20 to 25 degrees F. to about an average of 2^{o}F. Power and control systems are also discussed.

Rajic presents in his article Refrigeration Techniques and Thermoelectric Phenomena (18), the theoretical bases of analyzing thermo-electric phenomena, and of the technical calculations of thermo-electric refrigeration devices. To determine characteristics of thermo-electric elements, scientists have suggested various coefficients, such as: "effective thermal force" (E. Altenkirch), dimensionless coefficient θ (H. J. Goldsmid, R. W. Douglas) and "effectiveness of thermoelement" Z (Ioffe). $Z = \alpha^2 \sigma / \lambda$

where α is the coefficient of proportionality of the thermo-electric exciting force according to Zebeke; $E = \alpha (T - T_0)$ where E is the voltage, T and T_0 - the temperature of the hot and cold junction respectively; σ - the specific electric conductivity of the thermo-electric element, and λ is the coefficient of its heat conductivity. If these characteristics are known, it is easy to determine the maximum value of the hot and cold junction temperature $(T/T_0)_{max}$ for obtaining the maximum value of operation current in the circuit, $I_{optim.}$. The efficiency of the refrigerating unit is determined by the refrigeration coefficient $\mathcal{E}_{max}$ found from equation $\mathcal{E}_{max} = T/(T-T_0) = [\sqrt{1 + Z(T+T_0)/2} - T/T_0]/[\sqrt{1 + Z(T+T_0)/2} + 1]$. Data are given on the selection of materials for thermo-electric elements, in particular, some characteristics of semiconductor alloys Bi_2Te_2 (52.8% Bi + 47.8% Te) and Sb_2Te_3 (38.9% Sb + 61.1% Te) which were studied at the Institut Poluprovodnikov AN USSR (Institute of Semiconductors, AS USSR). The conclusions indicate advantages of thermo-electric refrigeration over the classical liquid one. For further successful utilization of thermo-electric phenomena in refrigeration engineering, joint efforts of physicists, chemists and metallurgists are imperative.

A paper by R. C. Miller et al. (14) theoretically analyzes the dependence of the figure of merit (Z) of thermoelectric material on the energy bandwidth. It was found that Z approaches zero as the bandwidth goes to zero. The existence of an optimin bandwidth is established. However the magnitude could not be ascertained.

A paper by Roland W. Ure, Jr. (21) presents comments on thermal conductivity in two-phase alloys. Quantitative agreement between the predictions of the model used and experimental data were obtained.

A paper by I. N. Pomazanov and P. L. Tikhomikov (Russia) (16) discusses the operation of a thermoelectric/generator/refrigerator operating from a gas burner. With a power output of from 2 to 3KW, sufficient refrigeration capacity was obtained to freeze about 0.7 kg (1.54 lbs.) of water per hour. One operational disadvantage of the system described was the high rate of flow of cooling water (100-120 liters/hr.) (26.4-31.7 gal./hr.).

In a report to the Department of the Navy (23) Carrier Corporation research work on thermoelectric air coolers and water chillers are described. Exact differential equations were written for heat and mass transfer relationships and these equations along with basic thermoelectric equations were solved by an analog computer.

In these analyses fixed cooling compartment temperatures were assumed. Figure of merit of thermoelectric materials ranged from $2.75 \times 10^{-3}/^{o}C$ to $3.04 \times 10^{-3}/^{o}C$. An air cooler of about 12,000 Btu/hr. capacity was designed and tested.

A report by the Whirlpool Corporation (24) to the Department of the Navy describes the design of a specific prototype, including controls, of a thermoelectric refrigerating system for use aboard submarines. One room is held at $0^{o}F$ and another can be held at $33^{o}F$ or $0^{o}F$ as may be required. It was assumed that thermoelectric materials would have a figure of merit of at least $2.5 \times 10^{-3}/^{o}C$. It was recommended from this study that a prototype refrigeration system be assembled and tested.

A report by the York Corporation to the Department of the Navy (25) includes a thorough study of present food preservation and storage practices. New food preservation methods such as freeze dehydration are discussed and revised food storage tables were developed. Another part of this report presents a preliminary study of the application of thermoelectricity to submarine food storage facilities.

Comments by W. Maurice Pritchard in the Proceedings of the IEEE (17), refer to previous work on calculating and optimizing the coefficient of performance of thermoelectric cooling devices. Previous studies have been based on the average thermoelectric properties and have neglected the Thomson effect. Also the effects of junction resistance and a.c. ripple in the power supply are important factors to be considered. The writer presents a mathematical approach to the optimization problem.

A status report by Dr. J. W. Davisson and Joseph Pasternak (5) presents thermoelectric materials data. The authors point out that no dramatic gains in the figure of merit have been accomplished at the time of their report. However they state that Bell Laboratories have developed a bismuth antimony alloy with a Z of better than 5×10^{-3} at 80 K. This is a very high Z and is a usable performance at cryogenic temperatures.

A paper by Marvin A. Fuller (9) discusses the mass production techniques of thermoelectric modules. He points out that a simple flat configuration which can be sandwiched between two heat sinks is desirable. Junction resistance problems are discussed along with the process apability level.

A report by Whirlpool Corporation to the Department of the Navy (26) describes a design study to optimize the application of a thermoelectric device for refrigeration aboard submarines. The study clearly showed that it is possible and practical to thermoelectrically provide such refrigeration in food storage rooms.

THEORETICAL ANALYSIS

Dimensional Analysis

The variables which influence a physical system can often be correlated by methods of dimensional analysis. Such an analysis offers a means of simplification by combining these factors into dimensionless groups which can facilitate experimental research.

Dimensional analysis establishes the relationship of these variables in a model which can then be compared with a prototype device.

To utilize dimensional analysis the variables considered to be important to the physical system are first selected. Then these variables are grouped into dimensionless "π" terms.

The "π" terms can then be related by an equation of the form:

$$\pi_1 = f(\pi_2, \pi_3 \ldots . \pi_n)$$

In order to determine the general equation the nature of the function "f" must be established. Satisfactory predictions can be made if the theoretical model and prototype are operated so that the values of the "π" terms in each case are equal.

The method of evaluating the function "f" is to hold one of the independent "π" terms constant and vary the other. This method is then applied to each of the "π" terms and the resulting component equations can be combined to give the general relationship. The final equation may be a polynomial type or exponential type depending on the relationship of the "π" terms.

Dimensionless Groups and "π" Terms

The design of this investigation was developed around the principles of dimensional analysis. The formation of dimensionless groups was investigated with an IBM 1620 computer to provide theoretical data. This data was supplied to an IBM 1410 computer which was programmed to statistically analyze the data. The output from the 1410 computer yielded a general prediction equation which could be experimentally investigated.

In analyzing the dimensionless "π" groups the following variables were considered to be pertinent:

Factor	Description	Dimension Mass-Length-Time Temperature Units
Q_c	Cooling capacity	ML^2T^{-3}
P_{wb}	Power required	ML^2T^{-3}
Δt_i	Difference between inlet water temperatures	θ
Δt_c	Drop in water temperature due to cooling	θ
W_c	Water flow rate through cooling sink	MT^{-1}
W_h	Water flow rate through heating sink	MT^{-1}

The required number of "π" terms can be determined as follows:

$$P_{wb} = K(Q_c)^a, (\Delta t_i)^b, (\Delta t_c)^c, (W_c)^d, (W_h)^e \tag{1}$$

Substituting the dimensions for each term:

$$\frac{ML^2}{T^3} = \left(\frac{ML^2}{T^3}\right)^a, \theta^b, \theta^c, \left(\frac{M}{T}\right)^d, \left(\frac{M}{T}\right)^e \tag{2}$$

Equations can then be written by summarizing the exponents for each dimension as follows:

$$\Sigma M: \quad 1 = a + d + e \tag{3}$$

$$\Sigma L: \quad 2 = 2a \tag{4}$$

$$\Sigma T: \quad -3 = -3a - d - e \tag{5}$$

$$\Sigma \theta: \quad 0 = b + c \tag{6}$$

From these equations the following is obtained:

$$a = 1$$

$$d + e = 0 \quad \text{or} \quad d = -e$$

$$c = -b \quad \text{or} \quad b = -c$$

Substitution of the above values into equation (2) yields:

$$P_{wb} = K(Q_c), \left(\frac{\Delta t_c}{\Delta t_i}\right)^c, \left(\frac{W_c}{W_h}\right)^d \tag{7}$$

From equation (7) by rearranging terms the following results:

$$\frac{Q_c}{P_{wb}} = K\left(\frac{\Delta t_i}{\Delta t_c}\right)^b \left(\frac{W_c}{W_h}\right)^d \tag{8}$$

The relationship between the three "π" terms thus formed can be analyzed by holding one constant while the other two are varied. The analysis was developed as follows:

First set of data from computer:

$$\pi_1 = f(\pi_2, \overline{\pi_3}) \quad \text{where;} \quad (\pi_3 \text{ constant})$$

$$\pi_1 = \frac{Q_c}{P_{wb}}, \quad \pi_2 = \frac{\Delta t_i}{\Delta t_c}, \quad \pi_3 = \frac{W_c}{W_h}$$

Second set of data from computer:

$$\pi_1 = f(\pi_3, \overline{\pi}_2) \qquad (\pi_2 \text{ constant})$$

In the analysis another "π" term was used as a level indicator to correlate differences in cooling capacity and water inlet temperatures on the cooling side. The fourth "π" term was as follows:

$$\pi_4 = \frac{T_{wic}}{t_c} = \frac{\text{inlet water temperature on cooling side}}{\text{drop in water temperature due to cooling}}$$

This "π" term was held constant at four different values of inlet water temperature while Δt_c was held at a constant value of $2^{\circ}F$. The relationship between the other "π" terms was then determined at these levels. The data were then combined to yield a general prediction equation. A family of equations of surfaces can be obtained for various values of Δt_c. Engineering design can begin by selecting a value of Δt_c and then using the general prediction equation the other pertinent variables can be related.

Calculations to Determine Theoretical Equation for Heat Transfer Across the Heat Sinks

Basic Equation:

$$q = k\, A \frac{\Delta t}{\Delta x}$$

Values of Thermal Conductivity - "k"

Material	$\frac{\text{Btu}}{\text{hr.}\cdot\text{ft.}^2\cdot{}^\circ\text{F./ft.}}$	$\frac{\text{Gm. Cal.}}{\text{sec.}\cdot\text{cm.}^2\cdot{}^\circ\text{C./cm.}}$
Copper	222.0	0.918 (1)
Nylon (Monofilament)	0.145	0.0006 (2)
Silicone Grease Dow Corning # 340	0.363	0.0015 (3)

(1) At 18°C. - Handbook of Chemistry and Physics, Twenty Fifth Edition.

(2) At 68°F. - Elements of Material Science, by VanVlack. Second Edition by Addison Wesley, 1964, p. 420.

(3) Dow Corning Corporation, Bulletin # 04-027, September 1963.

Figure 1 shows the cross sectional view of the heat sink copper plate and the nylon/grease separator between the heat sink and the thermoelectric module. The total area considered in these calculations is one square foot. 100 mesh nylon material has an open area of approximately 36 percent.* Therefore the nylon occupies 64 percent of the area and the silicone grease fills the remaining 36 percent.

Assuming a ten degree F. temperature difference across the nylon and grease separator the following calculations are considered:

$$\text{heat transferred} = \frac{\text{thermal conductivity} \times \text{percent of area} \times \text{temperature difference}}{\text{thickness of nylon/grease separator}}$$

*Tobler, Ernst & Traber, Inc., Price List N-7, May 1962.

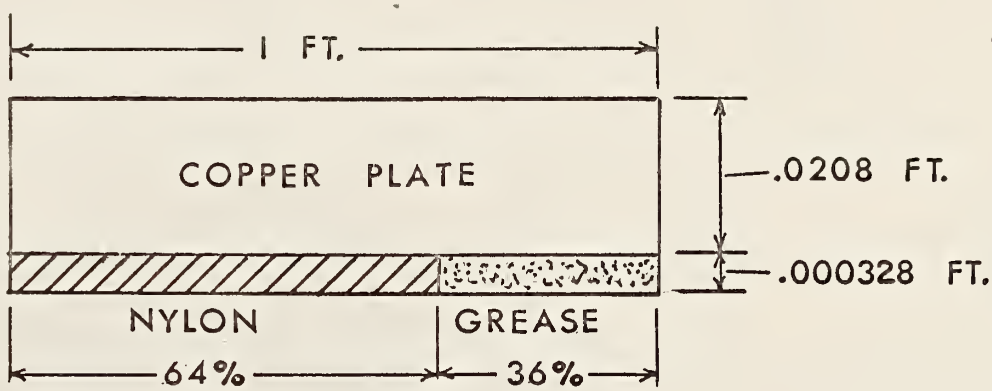

Fig. 1. Cross section of heat sink copper plate and nylon/grease separator.

or:

$$q = \frac{(k)\ (A)\ (\Delta t)}{\Delta x_{ng}} \quad \text{Btu./Hr.} \tag{9}$$

where Δx_{ng} is determined as follows;

100 mesh nylon monofilament is approximately 100 microns* in diameter. And,

$$1 \text{ micron } (10^{6}) = 1 \text{ meter} \quad \text{and} \quad 1 \text{ meter} = 39.37 \text{ inches.}$$

$$\text{Therefore, } 100 \text{ microns} = 10^{-4} \text{ meters} = \frac{(10^{-4})\ (39.37)}{12}$$

$$\Delta x_{ng} = 0.000328 \text{ feet}$$

$$\text{therefore; } q(\text{Nylon}) = \frac{(0.145)\ (0.64)\ (10)}{0.000328} = 2829 \text{ Btu./Hr.} \tag{10}$$

$$\text{and; } \quad q(\text{Grease}) = \frac{(0.363)\ (0.36)\ (10)}{0.000328} = 3984 \text{ Btu./Hr.} \tag{11}$$

$$q(\text{nylon}) + q(\text{grease}) = 6813 = \frac{\bar{k}_{ng}\ A\ \Delta t}{\Delta x_{ng}} = \frac{\bar{k}\ (1)\ (10)}{0.000328} \tag{12}$$

$$\text{therefore; } \quad \bar{k}_{ng} = 0.2235 \text{ Btu./hr.} \cdot {}^{o}\text{F.} \cdot \text{Ft.}^{2} \text{ per foot.}$$

$$\text{and; } \quad R(\text{nylon/grease}) = \frac{\Delta x(\text{nylon/grease})}{\bar{k}_{ng}} = \frac{0.000328}{0.2235} \tag{13}$$

$$= 0.00147 \text{ ft.}^{2} \cdot \text{hr.} \cdot {}^{o}\text{F./Btu.}$$

$$R(\text{copper plate}) = \frac{\Delta x(\text{copper plate})}{k_{cu.}} = \frac{0.0208}{222} \tag{14}$$

$$= 0.0000937 \text{ ft.}^{2} \cdot \text{hr.} \cdot {}^{o}\text{F./Btu.}$$

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*Tobler, Ernst & Traber, Inc., Price List N-7, May 1962.

$$R = 0.00147 + 0.0000937 = 0.0015637 \tag{15}$$

and

$$\bar{C} = \frac{1}{R} = \frac{1}{0.0015637} = 639.5 \text{ Btu./ft.}^2\cdot\text{hr.}\cdot{}^{\circ}\text{F.} \tag{16}$$

therefore;

$$q = \bar{C}\ A\ \Delta t = (639.5)\ (0.0273)\ (\Delta t) = 17.46\ \Delta t \quad \text{Btu./hr.}) \tag{17}$$

Equation (17) is shown graphically in Fig. 2.

Theoretical Flat Plate Heat Transfer Calculations

Empirical Equation:

$$N_{Nu} = \frac{h\ L}{k} = 0.664\ (N_{Re})^{\frac{1}{2}}\ (N_{Pr})^{1/3\,*} \tag{18}$$

Calculation of Reynolds Number:**

$N_{Re} = \frac{L\ v}{\nu}$ where; L = length of plate in feet

v = velocity of fluid in ft./hr.

ν = kinematic viscosity in ft.^{2}/hr.

Given: ν = 0.0298 ft.^{2}/hr. @ 90°F.

L = 0.219 ft., length of plate (see Fig. 3).

m = 10 lbs./hr., fluid flow rate.

t = 90°F. water inlet temperature.

A = 0.0026 ft.^{2}, cross-sectional area (see Fig. 3).

ρ = 62.1 lb_m/ft.^{3}, water density @ 90°F.

k = 0.354 Btu./hr.·ft.·$^{\circ}$F., @ 90°F.

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*For forced convection, laminar flow.

**Sample calculations.

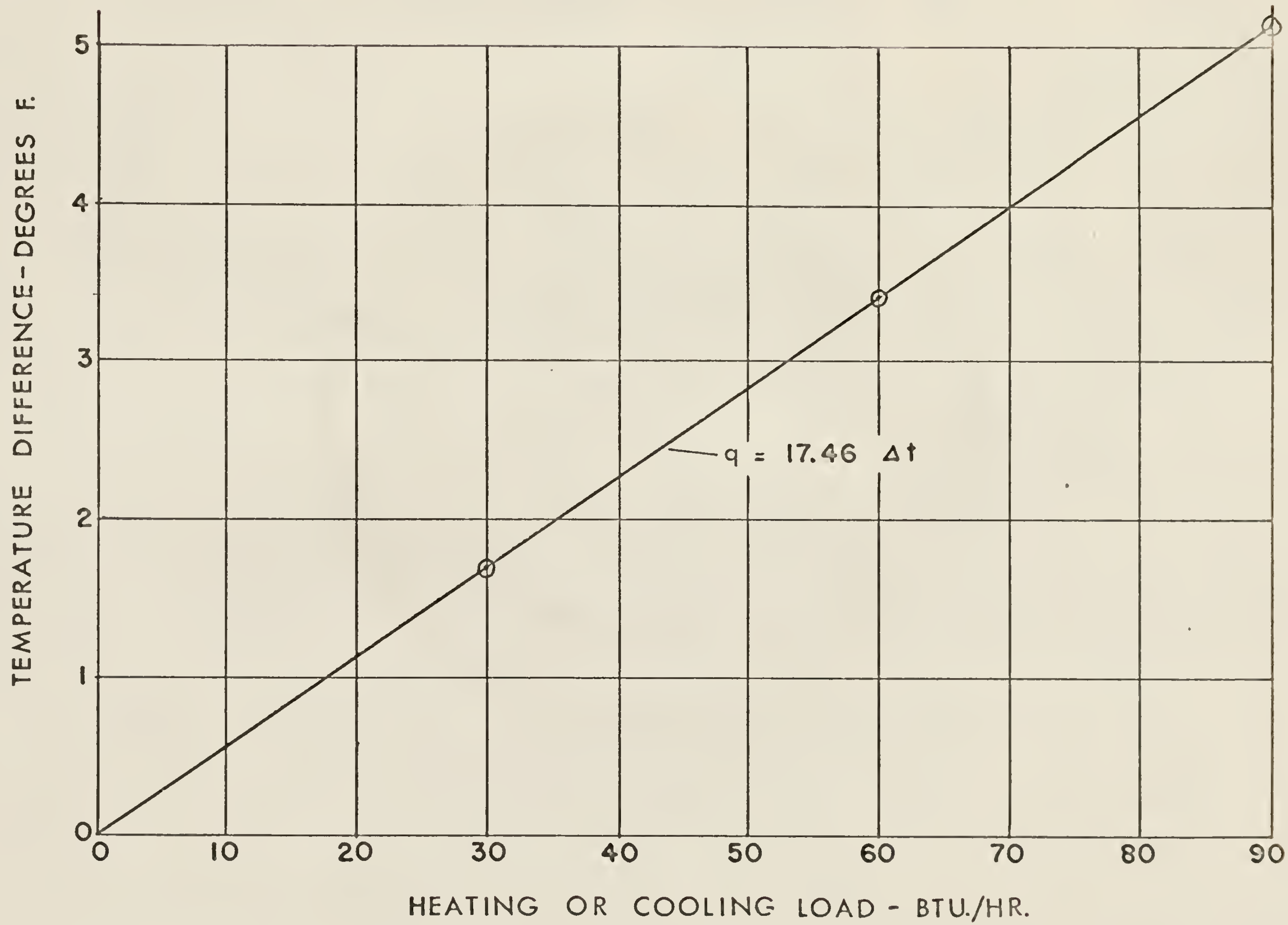


Fig. 2. Plot of theoretical temperature difference between heat sinks and thermoelectric module junctions vs. heating or cooling load where "Δt" is in degrees F. and "q" is in Btu./Hr.

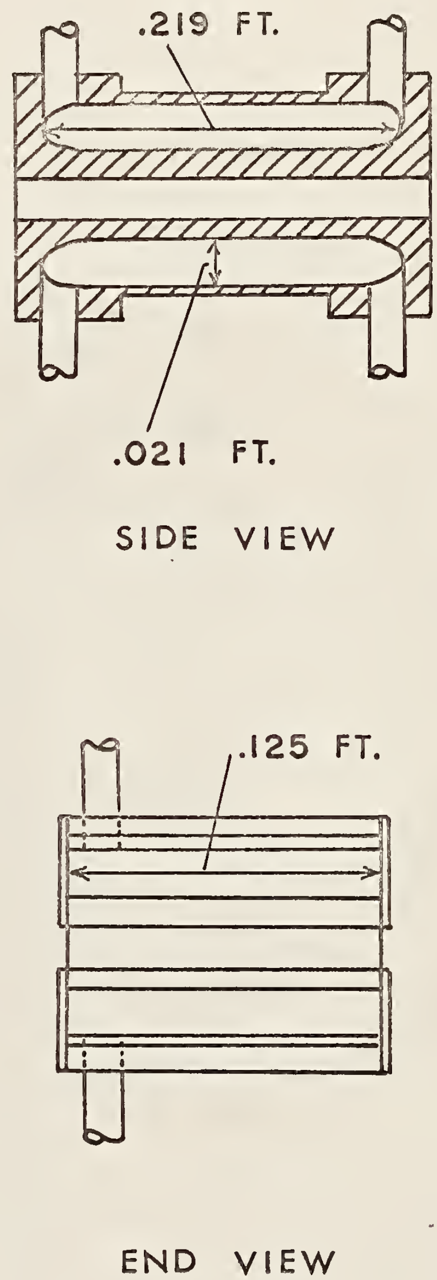


Fig. 3. Dimensions used for theoretical flat plate heat transfer calculations.

then;

$$\text{Volume of water flowing ft.}^3\text{/sec.} = V = \frac{\text{lbs./hr.}}{(3600)\ (\text{lb}_m/\text{ft.}^3} = \frac{10}{(3600)\ (62.1)} \qquad (19)$$

$$V = 0.00004477 \text{ ft.}^3\text{/sec.}$$

and;

$$\text{Velocity of water flowing ft./hr.} = v = \frac{\text{ft.}^3\text{/sec. (3600)}}{\text{cross-sectional area}} = \frac{0.00004477\ (3600)}{0.0026} \qquad (20)$$

$$v = 61.9 \text{ ft./hr.}$$

then;

$$N_{Re} = \frac{L\ v}{\nu} = \frac{(0.219)\ (61.9)}{0.0298} = 454.9 \qquad (21)$$

and;

$$N_{Pr} = 5.23$$

<u>Calculation of Heat Transfer Coefficient</u>:**

$$N_{Nu} = 0.664\ (N_{Re})^{\frac{1}{2}}\ (N_{Pr})^{1/3} \qquad (22)$$

$$= 0.664\ (454.9)^{\frac{1}{2}}\ (5.23)^{1/3} = 0.664$$

$$= 24.59$$

then;

$$h = \frac{(N_{Nu})\ (k)}{L} = \frac{(24.59)\ (0.354)}{0.219} \qquad (23)$$

$$h = 39.75 \text{ Btu./hr.}\cdot\text{ft.}^{2}\cdot{}^{o}\text{F.}$$

**Sample calculations.

The theoretical values of the heat transfer coefficient are shown in Table 1. These values are shown graphically in Fig. 4.

Theoretical Heat Transfer Data Plotted on Logarithmic Scales

The data obtained from the theoretical analysis of flat plate heat transfer were plotted on logarithmic scales. Graphs of equal water temperatures (60°F., 75°F., 90°F.) were obtained by plotting the heat transfer coefficient versus the mass flow rate. These data plotted as straight lines on logarithmic coordinates as shown in Fig. 5.

From this data equations for each water temperature line were obtained as follows;

Equation is of the form: $y = bx^m$

<u>Determination of the slope</u>:

$$m = \frac{\log y_2 - \log y_1}{\log x_2 - \log x_1} \quad : \text{ from calculations:}$$

$$y_1 = 39.75 \text{ and } x_1 = 10$$
$$y_2 = 68.83 \qquad x_2 = 30$$

$$m = \frac{\log 68.83 - \log 39.75}{\log 30 - \log 10}$$

$$m = \frac{1.83778 - 1.59934}{1.47712 - 1.0} = 0.49974 \tag{24}$$

Determination of "y" Intercept:

$$y = bx^m \tag{25}$$

Therefore: $b = \frac{y}{x^m}$ or $\log b = \log y - m\log x$

And for $x = 10$

Table 1. Data used to calculate the theoretical heat transfer coefficient for a flat copper plate.

Variable Factor	Mass flow of water and water temperature								
	10 lbs./hr.			20 lbs./hr.			30 lbs./hr.		
	60^{o}F.	75^{o}F.	90^{o}F.	60^{o}F.	75^{o}F.	90^{o}F.	60^{o}F.	75^{o}F.	90^{o}F.
Volume of water– ft.^{3}/sec. (x 10^{-6})	44.55	44.62	44.77	89.10	98.24	89.54	133.65	133.87	134.31
Velocity of water– ft./hr.	61.7	61.79	61.9	123.40	123.57	123.80	185.10	185.36	185.70
Reynolds Number N_{Re} (Dimensionless)	311.3	380.09	454.9	622.60	760.17	909.80	933.90	1140.26	1364.70
Nusselt Number N_{Nu} (Dimensionless)	23.47	24.03	24.59	33.18	33.99	34.77	40.64	41.63	42.58
Heat Transfer Coefficient--"h" Btu./hr.·ft.^{2}·^{o}F.	36.12	37.98	39.75	51.06	53.71	56.20	62.54	65.78	68.83

Table 1 (Cont.).

Variable Factor	Mass flow of water and water temperature: 40 lbs./hr.			50 lbs./hr.		
	60^{o}F.	75^{o}F.	90^{o}F.	60^{o}F.	75^{o}F.	90^{o}F.
Volume of water- ft.^{3}/sec. (x 10^{-6}	178.2	178.49	179.08	222.75	223.11	223.85
Velocity of water- ft./hr.	246.8	247.14	247.6	308.5	308.93	309.5
Reynolds Number N_{Re} (Dimensionless)	1245.2	1520.34	1819.6	1556.5	1900.43	2274.5
Nusselt Number N_{Nu} (Dimensionless)	46.94	48.08	49.16	52.48	53.75	54.97
Heat Transfer Coefficient--"h" Btu./hr.·ft.^{2}·^{o}F.	72.23	75.96	79.46	80.76	84.93	88.86

Other Factors: Density (lb_m/ft.^{3}) = 62.4 (60^{o}F.), 62.25 (75^{o}F.), 62.1 (90^{o}F.)

Kinematic Viscosity (ft.^{2}/hr.) = 0.0434 (60^{o}F.), 0.0356 (75^{o}F.), 0.0298 (90^{o}F.)

Thermal Conductivity (Btu/hr. ft.^{2o}F) = 0.337 (60^{o}F.), 0.346 (75^{o}F), 0.354 (90^{o}F.)

Prandtl Number (N_{PR} Dimensionless) = 8.04 (60^{o}F.), 6.40 (75^{o}F.), 5.23 (90^{o}F.)

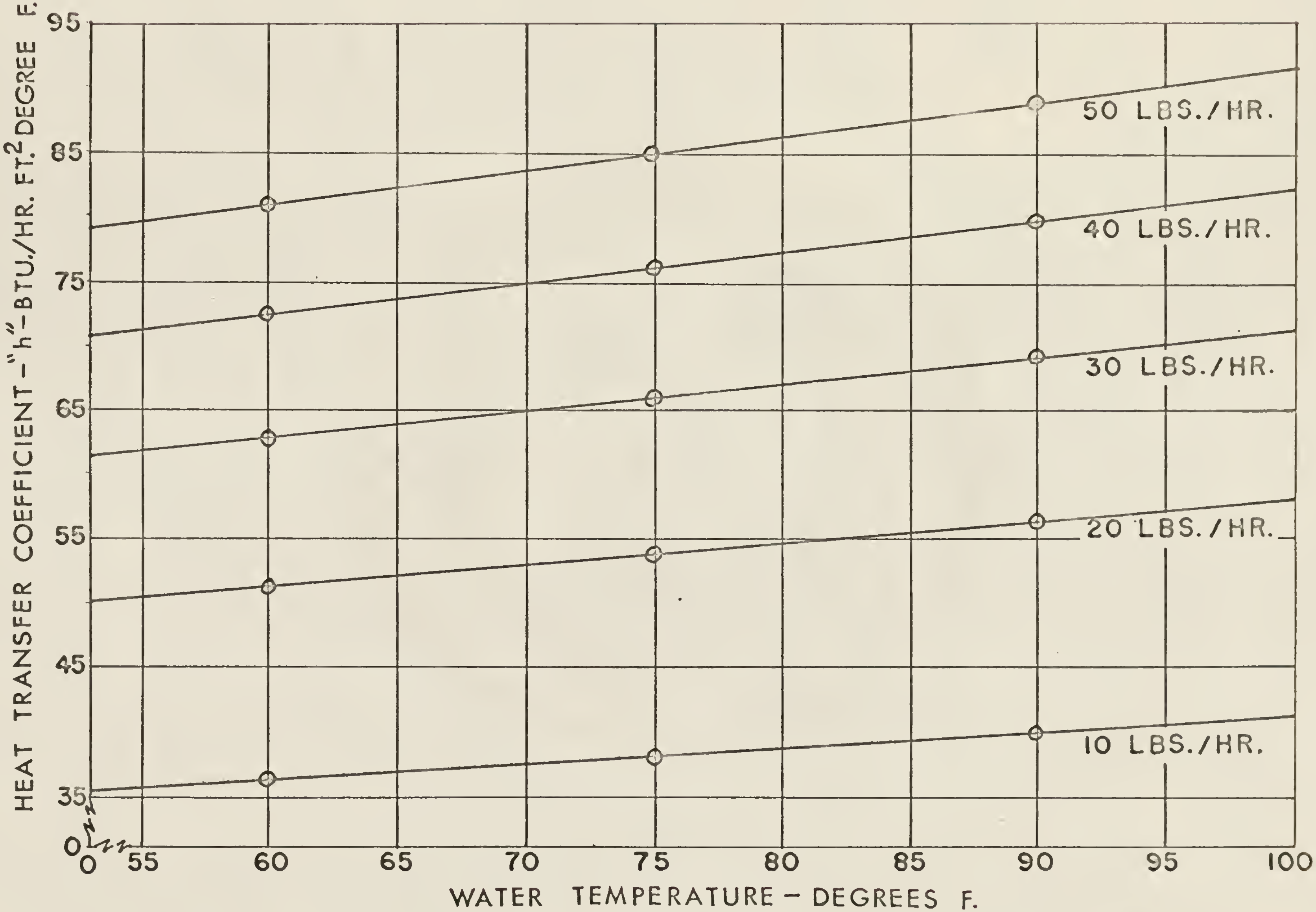


Fig. 4. Theoretical heat transfer coefficient "h" versus water temperature and mass rate of flow for a flat copper plate.

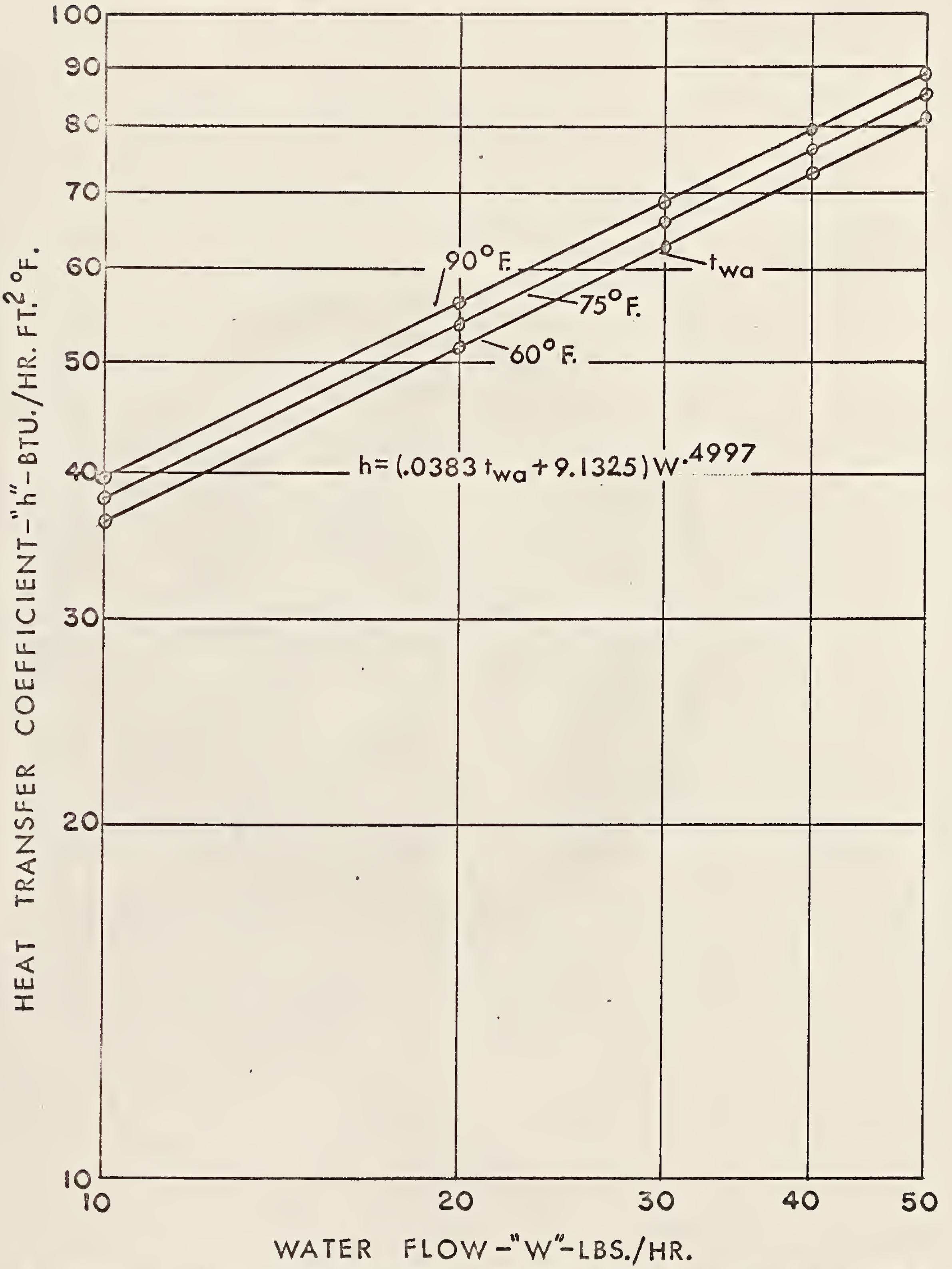


Fig. 5. Plot of heat transfer coefficient "h" versus mass flow rate "W" for water temperatures of 60°F., 75°F. and 90°F. on logarithmic coordinates.

$$\log b = \log 39.75 - 0.49974 \log 10$$

$$\log b = 1.59934 - 0.49974$$

$$\log b = 1.09960$$

Therefore: $b = 12.58$ ("y" intercept)

Equation of the line, heat transfer coefficient vs. flow rate for water temperature of 90°F. is:

$$y = 12.58\ x^{0.4997}$$

Similarly:

Equation of the line, heat transfer vs. flow rate for a water temperature of 60°F. is:

$$y = 11.43\ x^{0.4997}$$

Determination of General Equation for Theoretical Heat Transfer Coefficient

From previous calculations it was determined that the slope of each water temperature line was approximately equal. The temperature of the water caused the lines to move vertically on the graph.

The calculations also showed that they intercept changes in equal segments for each equal segment of change in water temperature. Therefore the following relationship was noted:

For 90° water temperature b_1 (y intercept) is 12.58.

For 60° water temperature b_2 (y intercept) is 11.43.

and; $\Delta b = b_1 - b_2 = 12.58 - 11.43 = 1.15$

Dividing Δb by 30 yields a value of "b" for each degree change in water temperature.

and; $\frac{\Delta b}{30} = \frac{1.15}{30} = 0.0383$

Therefore b_{61^oF} would be 11.4683; b_{62^oF} would be 11.5066, etc. If the above were true from 0^oF to 60^oF then multiplying the water temperature by .0383 would yield the correct b (y intercept). However the following was noted:

(60) (.0383) = 2.2980 (for 60^oF water temperature)

Subtracting 2.2980 from 11.43 (b_{60^oF}) = 9.132

Similarly:

(90) (.0383) = 3.4470 (for 90^oF water temperature)

Subtracting 3.4470 from 12.58 (b_{90^oF}) = 9.133

Therefore the constant 9.1325 must be added to each water temperature multiplied by 0.0383.

The mathematical relationship was determined to be;

b = (0.0383) (water temperature) + 9.1325

The general equation used to relate water flow, average water temperature and the heat transfer coefficient was;

$$y = (0.0382\ t_{wa} + 9.1325)\ x^{.4997} \qquad (26)$$

where;

y = heat transfer coefficient - Btu./hr.ft.^{2} ^oF.

t_{wa} = average water temperature - ^oF.

x = water flow rate - lbs./hr.

Theoretical Dimensional Analysis Calculations

Assumptions (π_3 Constant):

$Twic = 90^{o}F.$

$\Delta t_c = 2^{o}F.$

$$\pi_4 = \frac{Twic}{\Delta t_c} = \frac{90}{2} = 45 \text{ (constant)}$$

$$\pi_3 = \frac{W_c}{W_h} = 0.5 \text{ (constant)}$$

$W_c = 10$ lbs./hr.

Total cooling capacity;

$$Q_c = (W_c)(\Delta t_c) = (10)(2) = 20 \text{ Btu./hr.} \qquad (27)$$

Cooling capacity per couple;

$$q_c = \frac{Q_c}{(12)(3.413)} = \frac{20}{(12)(3.413)} = 0.488 \text{ watts} \qquad (28)$$

Outlet water temperature on cooling side;

$$Twoc = Twic - \Delta t_c = 88\ ^{o}F. \qquad (29)$$

Temperature of plate on cold side;

From previous heat transfer relationships it was shown that;

$$Q_c = h\ A\ \Delta t_{ln} \quad \text{or} \quad h = \frac{Q_c}{A\ \Delta t_{ln}} \qquad (30)$$

Also,

$$y = (.0383\ t_{wa} + 9.1325)\ x^{.4997} \text{ (Equation 26)} \qquad (31)$$

Also it is known that,

$$h \equiv y$$

Therefore,

$$\frac{Q_c}{A\,\Delta t_{ln}} = (.0383\ t_{wa} + 9.1325)\ x^{.4997} \tag{32}$$

where,

$$\Delta t_{ln} = \frac{\Delta t_c}{\ln \dfrac{(Twic - t_{pc})}{(Twoc - t_{pc})}}$$

and,

$$x = W_c \quad ; \quad t_{wa} = t_{pc} + \Delta t_{ln} \quad ; \quad Twoc = Twic - \Delta t_c$$

therefore by trial and error substitution for t_{pc} equation (32)and be solved and the solution is;

$$t_{pc} = 70.72\ {}^{o}F.^{*}$$

Temperature gradient across nylon, grease and copper plate;

$$\Delta t_{cg} = \frac{Q_c}{17.46} = \frac{20}{17.46} = 1.1455\ {}^{o}F. \tag{33}$$

Temperature of cold junction;

$$t_{cf} = t_{pc} - \Delta t_{cg} = 69.5745\ {}^{o}F.^{*} \tag{34}$$

$$t_{cc} = 5/9\ (t_{cf} - 32) = 20.8747\ {}^{o}C.$$

---

*IBM 1620 computer data.

Temperature difference between hot and cold junctions;

$$\Delta t_{hc} = \frac{1.13 + .00678(t_{cc}) - q_c}{.0176} = 44.5001\ ^{o}C.^{*} \tag{35}$$

Temperature of hot junctions;

$$t_{hc} = t_{cc} + \Delta t_{hc} = 65.3748\ ^{o}C. \tag{36}$$

$$t_{hf} = (9/5)\ (t_{hc}) + 32 = 149.6747\ ^{o}F.^{*}$$

Millivolt drop across each couple;

$$V_m = .526\ (\Delta t_{hc}) + 43.4 = 66.8071\ MV. \tag{37}$$

Total power required for module;

$$P_w = \frac{[V_m + (.017)\ (I)\ (t_{cc})]\ [(12)\ (I)]}{1000} = 12.9834\ \text{watts}^{*} \tag{38}$$

$$P_{wb} = (P_w)\ (3.413) = 44.3124\ \text{Btu./hr.}^{*}$$

Total heat to be rejected at hot junctions;

$$Q_h = Q_c + P_{wb} = 20 + 44.3124 \quad 64.3124\ \text{Btu./hr.}^{*} \tag{39}$$

Temperature gradient across nylon, grease and copper plate;

$$\Delta t_{cg} = \frac{Q_h}{17.46} = 3.6834\ ^{o}F. \tag{40}$$

Temperature of plate on hot side;

$$t_{ph} = t_{hf} - \Delta t_{cg} = 145.9913\ ^{o}F.^{*} \tag{41}$$

Water flow rate through hot side;

$$W_h = \frac{W_c}{P_3} = \frac{10}{.5} = 20\ \text{lbs./hr.} \tag{42}$$

Temperature rise of water through hot side;

$$\Delta t_h = \frac{Q_h}{W_h} = 3.2156\ ^{o}F.^{*} \tag{43}$$

Inlet water temperature at hot side;

From previous heat transfer relationships,

$$Q_h = h\ A\ \Delta t_{ln} \quad \text{or} \quad h = \frac{Q_h}{A\ \Delta t_{ln}} \tag{44}$$

and,

$$y = (.0383\ t_{wa} + 9.1325)\ x^{.4997} \quad \text{(Equation 26)} \tag{45}$$

and it is known that,

$$h \equiv y$$

where,

$$\Delta t_{ln} = \frac{\Delta t_h}{\ln\dfrac{(Twih - t_{ph})}{(Twoh - t_{ph})}}$$

and,

$$x = W_h \quad ; \quad t_{wa} = t_{ph} - \Delta t_{ln}$$

therefore,

$$\frac{Q_h}{A\ \Delta t_{ln}} = (.0383\ t_{wa} + 9.1325)\ x^{.4997} \tag{46}$$

and by trial and error substitution for Twih equation (46) can be solved and the solution is,

$$Twih = 104.7413\ ^{o}F.^{*}$$

Outlet water temperature at hot side;

$$Twoh = Twih + \Delta t_h = 107.9769\ ^{o}F. \tag{47}$$

$$\pi_1 = \frac{Q_c}{P_{wb}} = \frac{20}{44.3124} = .4513^{*} \tag{48}$$

$$\pi_2 = \frac{Twih - Twic}{\Delta t_c} = \frac{104.7413 - 90.}{2} = 7.3707^{*} \tag{49}$$

$$\pi_3 = \frac{W_c}{W_h} = \frac{10}{20} = .5 \text{ (constant)} \tag{50}$$

$$\pi_4 = \frac{Twic}{\Delta t_c} = \frac{90}{2} = 45 \text{ (constant)} \tag{51}$$

Assumptions π_2 Constant:

$Twic = 90^{o}$ F.

$\Delta t_c = 2^{o}$ F.

$$\pi_4 = \frac{Twic}{\Delta t_c} = \frac{90}{2} = 45 \text{ (constant)}$$

$Twih = 110^{o}$ F.

$$\pi_2 = \frac{Twih - Twic}{\Delta t_c} = \frac{110 - 90}{2} = 10. \text{ (constant)}$$

$W_c = 10$ lbs./hr.

$A = .0273$ ft.^{2}

Calculations:

Total cooling capacity;

$$Q_c = (W_c)\ (\Delta t_c) = (10)\ (2) = 20 \text{ Btu./hr.} \tag{52}$$

Cooling capacity per couple;

$$q_c = \frac{Q_c}{(12)\ (3.413)} = 0.488 \text{ watts} \tag{53}$$

Outlet water temperature on cooling side;

$$\text{Twoc} = \text{Twic} - \Delta t_c = 90 - 2 = 88^{\circ}\ \text{F.} \tag{54}$$

Temperature of plate on cold side;

From previous heat transfer relationships it was shown that,

$$Q_c = h\ A\ \Delta t_{ln} \quad \text{or} \quad h = \frac{Q_c}{A\ \Delta t_{ln}} \tag{55}$$

Also,

$$y = (.0383\ t_{wa} + 9.132)\ x^{.4997} \quad \text{(Equation 26)} \tag{56}$$

Also it is known that,

$$h \equiv y$$

Therefore,

$$\frac{Q_c}{A\ \Delta t_{ln}} = (.0383\ t_{wa} + 9.132)\ x^{.4997} \tag{57}$$

where,

$$\Delta t_{ln} = \frac{\Delta t_c}{\ln \frac{(\text{Twic} - t_{pc})}{(\text{Twoc} - t_{pc})}}$$

and,

$$x = W_c \quad ; \quad t_{wa} = t_{pc} + \Delta t_{ln} \quad ; \quad \text{Twoc} = \text{Twic} - \Delta t_c$$

therefore by trial and error substitution for t_{pc} equation (57) can be solved and the solution is;

$$t_{pc} = 70.72\ ^{o}F.^{*}$$

Temperature gradient across nylon, grease and copper plate;

$$\Delta t_{cg} = \frac{Q_c}{17.46} = \frac{20}{17.46} = 1.1455\ ^{o}F. \tag{58}$$

Temperature of cold junctions;

$$t_{cf} = t_{pc} - \Delta t_{cg} = 69.5745\ ^{o}F.^{*} \tag{59}$$

$$t_{cc} = 5/9\ (t_{cf} - 32) = 20.8747\ ^{o}C. \tag{60}$$

Temperature difference between hot and cold junctions;

$$\Delta t_{hc} = \frac{1.13 + .00678\ (t_{cc}) - q_c}{.0176} = 44.5001\ ^{o}C.^{*} \tag{61}$$

Temperature of hot junction;

$$t_{hc} = t_{cc} + \Delta t_{hc} = 65.3748\ ^{o}C. \tag{62}$$

$$t_{hf} = (9/5)\ (t_{hc}) + 32 = 149.6747\ ^{o}F.^{*}$$

Millivolt drop across each couple;

$$V_m = .526\ (\Delta t_{hc}) + 43.4 = 66.8071\ MV. \tag{63}$$

Total power required for module;

$$P_w = \frac{\left[V_m + (.017)\ (I)\ (t_{cc})\right]\left[(12)\ (I)\right]}{1000} = 12.9834\ \text{watts}^{*} \tag{64}$$

---

*IBM 1620 computer data.

$$P_{wb} = (P_w)\ (3.413) = 44.3124 \text{ Btu./hr.}^* \tag{65}$$

Total heat to be rejected at hot junctions;

$$Q_h = Q_c + P_{wb} = 64.3124 \text{ Btu./hr.}^* \tag{66}$$

Temperature gradient across nylon, grease and copper plate;

$$\Delta t_{cg} = \frac{Q_h}{17.46} = 3.6834\ ^{o}\text{F.} \tag{67}$$

Temperature of plate on hot side;

$$t_{ph} = t_{hf} - \Delta t_{cg} = 145.9913\ ^{o}\text{F.}^* \tag{68}$$

Water flow through hot side;

From previous heat transfer relationships;

$$Q_h = h\ A\ \Delta t_{ln} \quad \text{or} \quad h = \frac{Q_h}{A\ \Delta t_{ln}} \tag{69}$$

and,

$$y = (.0383\ t_{wa} + 9.132)\ x^{.4997} \tag{70}$$

and it is known that,

$$h \equiv y$$

where,

$$\Delta t_{ln} = \frac{\Delta t_h}{\ln \dfrac{(Twih - t_{ph})}{(Twoh - t_{ph})}} \quad ; \quad x = W_h \quad ; \quad t_{wa} = t_{ph} - \Delta t_{ln}$$

and;

$$\frac{Q_h}{A\ \Delta t_{ln}} = (.0383\ t_{wa} + 9.132)\ x^{.4997} \tag{71}$$

therefore by trial and error substitution for W_h equation (71) can be solved and the solution is;

$$W_h = 25.35\ \text{lbs./hr.}^{*}$$

Temperature rise of water through hot side;

$$\Delta t_h = \frac{Q_h}{W_h} = 2.5370\ ^{o}F.^{*} \tag{72}$$

Outlet water temperature at hot side;

$$Twoh = Twih + \Delta t_h = 112.5370\ ^{o}F. \tag{73}$$

Therefore;

$$\pi_1 = \frac{Q_c}{P_{wb}} = .4513 \quad *(\text{variable}) \tag{74}$$

$$\pi_2 = \frac{Twih - Twic}{\Delta t_c} = 10.0 \quad *(\text{constant}) \tag{75}$$

$$\pi_3 = \frac{W_c}{W_h} = 0.3945 \quad *(\text{variable}) \tag{76}$$

$$\pi_4 = \frac{Twic}{\Delta t_c} = 45 \quad *(\text{constant}) \tag{77}$$

Analysis of Theoretical Data

The theoretical data acquired from the IBM 1620 computer were programmed for multiple-regression analysis by an IBM 1410 computer. Plates I and II graphically show the theoretical data. Appendix C shows the data furnished the IBM 1410 computer.

The following equations were used for the statistical analysis:

1. $\pi_1 = A + B\pi_2 + C\pi_3 + D\pi_4$
2. $\pi_1 = A + B\pi_2 + C\pi_3 + D\pi_4 + E\pi_2\pi_3\pi_4$
3. $\pi_1 = A + B\pi_2 + C\pi_3 + D\pi_4 + E\pi_2\pi_3\pi_4 + F\pi_4^2$
4. $\pi_1 = A + B\pi_2 + C\pi_3 + D\pi_2\pi_3\pi_4$
5. $\pi_1 = A + B\pi_2 + C\pi_3 + D\pi_4^2$
6. $\pi_1 = A + B\pi_2 + C\pi_3 + D\pi_4 + E\pi_4^2$
7. $\pi_1 = A + B\pi_2 + C\pi_3 + D\pi_2\pi_3\pi_4 + E\pi_4^2$

The data also were converted to natural logarithmic values. These values were also processed by the IBM 1410 computer for the best fit analysis of an equation of the form;

$$\log \pi_1 = A + B\log \pi_2 + C\log \pi_3 + D\log \pi_4 \tag{78}$$

From equation (78) an equation of exponential form can be derived as follows;

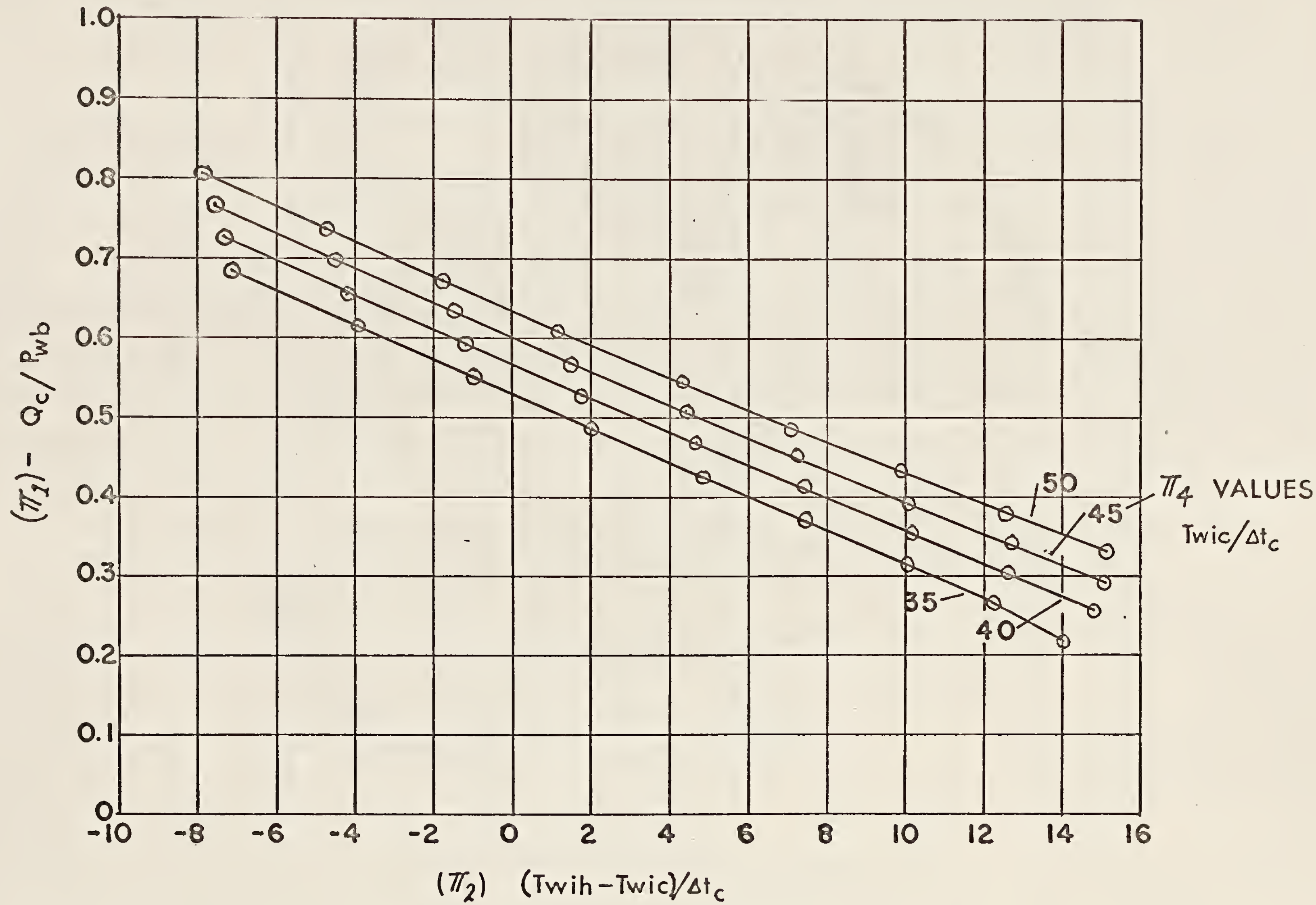
$$\pi_1 = e^A (\pi_2)^B (\pi_3)^C (\pi_4)^D \tag{79}$$

From the multiple regression analysis by the IBM 1410 computer the following polynomial equation was determined to provide adequate precision

EXPLANATION OF PLATE I

Plot of the computer data of the pi term Q_c/P_{wb} on the pi term $(T_{wih} - T_{wic})/\Delta t_c$ while holding the pi term W_c/W_h constant and the pi term $T_{wic}/\Delta t_c$ constant at four values.

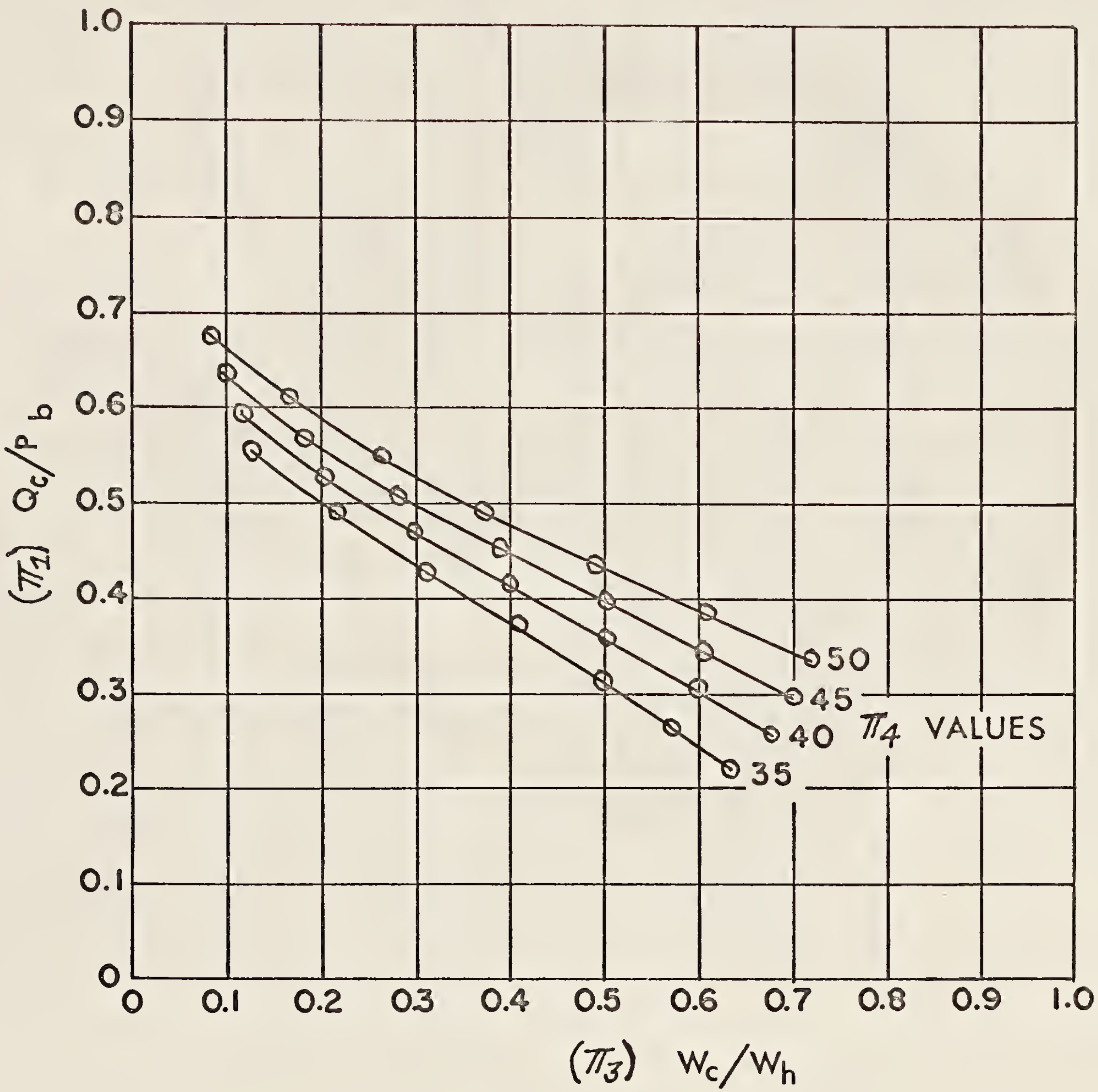
PLATE I


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
(π_1) - Q_c / P_{wb}
(π_2) (Twih-Twic)/Δt_c
π_4 VALUES
Twic/Δt_c
50
45
40
35

EXPLANATION OF PLATE II

Plot of the computer data of the pi term Q_c/P_{wb} on the pi term W_c/W_h, while holding the pi term $(T_{wih} - T_{wic})/\Delta t_c$ constant and the pi term $T_{wic}/\Delta t_c$ constant at four values.

PLATE II


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
(π_1) Q_c/P_b
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
(π_3) W_c/W_h
50
45
40
35
π_4 VALUES

for engineering design;

$$\pi_1 = 0.55162 - 0.02902\,\pi_2 - 0.56858\,\pi_3 + 0.00752\,\pi_4 \tag{80}$$

$$(R^2 = 0.996\ ,\ s_{y\cdot x} = 0.00945)$$

The exponential equation obtained from the computer was as follows;

$$\pi_1 = 0.091\,(\pi_2 + 10)^{-0.4333}\,(\pi_3)^{-0.3945}\,(\pi_4)^{0.6454} \tag{81}$$

$$(R^2 = 0.835\ ,\ s_{y\cdot x} = 0.13641)$$

Since π_2 had negative as well as positive values, the data for the logarithmic form had to be converted to positive values by adding a factor of 10 to each value of π_2. This explains the reason for $(\pi_2 + 10)$ appearing in the final equation.

The 1410 multiple regression computer program is not included in the thesis since it is a common item furnished by the computing center.

EXPERIMENTAL EQUIPMENT AND INSTRUMENTATION

The equipment used in this study consisted of the following:

1. A twelve-couple thermoelectric module of the type now commercially available.
2. Two flat-plate design heat sinks equipped for liquid flow.
3. A 2-foot square wooden box for housing the device and to provide for insulation of the device.
4. Two precisely controlled water baths, with stirrers, thermoregulators and overflow provision. The overflow was used to maintain a constant head on the device.
5. A vacuum tank for removing air from the distilled water.

6. A return tank and a supply tank with suitable pump.

7. A d.c. power supply consisting of two six volt automobile type batteries connected in parallel and two rheostats to control the current.

8. A thermocouple switch.

9. An electronic ice bath.

10. A precision potentiometer.

11. An ammeter and voltmeter.

12. A precision balance to batch weigh the liquid.

13. Needle valves to control flow rate.

14. Liquid filters and associated apparatus.

15. Calibrated thermocouples for determining the water temperatures and degrees of cooling and heating of the water by the device.

16. Thermocouples soldered to junctions to determine hot and cold junction temperatures.

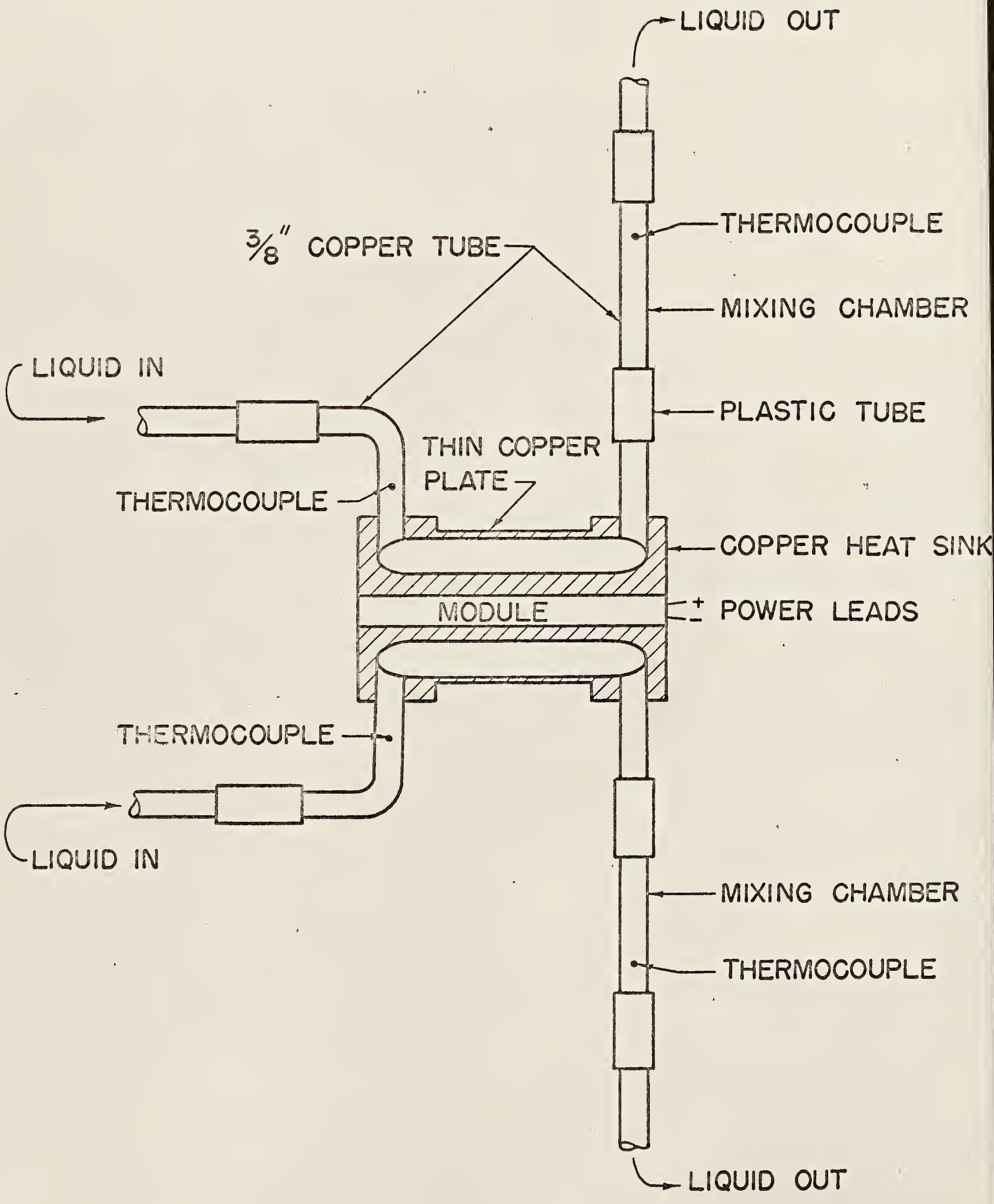
The liquid heat sinks were made of copper. The heat sinks were approximately 1.5 inches wide by 2.25 inches long so as to fit the heat transfer area of the thermoelectric module.

A mixing tube was attached to the outlet of each sink. The purpose of the mixing tube was to assure adequate mixing of the liquid before the outlet thermocouple sensed the liquid temperature. The mixing tube was made from one-half inch diameter copper tubing. Inside the tube and at one end a small copper disc was suspended by means of thin spiders in the center of the tube. Liquid flowing through the tube had to pass over and around this disc thus causing turbulence and subsequent mixing of the liquid. A cross-section of the device is shown in Plate III. Plate IV is a photograph of

EXPLANATION OF PLATE III

Cross-section of thermoelectric heating/cooling device showing arrangement of thermoelectric module, heat sinks inlets and outlets, thermocouples and associated equipment.

PLATE III


LIQUID OUT
THERMOCOUPLE
3/8" COPPER TUBE
MIXING CHAMBER
LIQUID IN
PLASTIC TUBE
THIN COPPER PLATE
THERMOCOUPLE
COPPER HEAT SINK
MODULE
± POWER LEADS
THERMOCOUPLE
LIQUID IN
MIXING CHAMBER
THERMOCOUPLE
LIQUID OUT

EXPLANATION OF PLATE IV

View of thermoelectric heating/cooling device mounted inside plywood container and ready for addition of insulation material.

PLATE IV

the device mounted inside the plywood container and ready for addition of the insulating material.

Short lengths of plastic tubing were used to connect the inlets and outlets to 3/8 inch diameter copper tubing which connected to the water bath outlets.

The block diagram of the laboratory equipment is shown in Plate V. The water baths were equipped with overflow outlets which provided the device with a constant head of water. By this system of overflows precise rates of flow were possible to attain and hold constant. Each water bath contained an electric stirrer and a precision mercury actuated thermoregulator. The mercury actuater was electrically connected to a solid-state, low current actuated, 5-ampere capacity relay. This relay controlled the power to the 300-watt low time lag water heaters in each water bath. Each water bath outlet was provided with a fiberglass water filter. Plate VI is a photograph of the experimental system as it appeared in the laboratory.

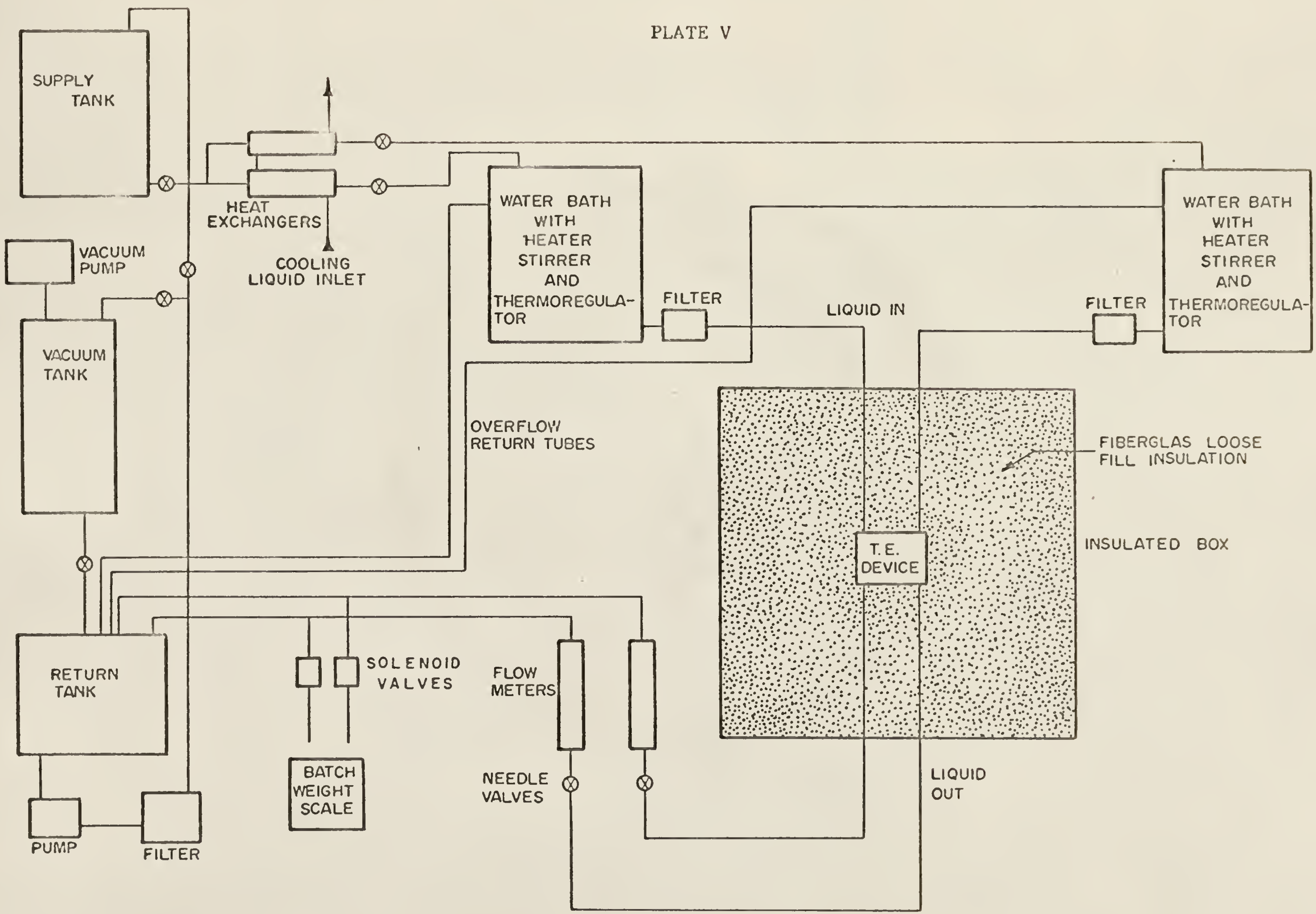
A twenty gallon plastic supply tank was provided to furnish the distilled water for both water baths. Water could be circulated through a heat exchanger before going into the water bath is a lower water bath temperature was required. Water entering the water baths was maintained several degrees cooler than the desired water bath temperature. The water bath heaters then heated the water to the desired temperature and turned off as determined by the thermoregulators. Cooling water was obtained from a standard electric water fountain type cooler.

A vacuum tank was constructed from a liquified gas cylinder. A vacuum pump was used to obtain a high vacuum (about 28 inches Hg.) on a portion of the distilled water which was in the tank. The vacuum pump was allowed to

EXPLANATION OF PLATE V

Block diagram of the laboratory equipment.

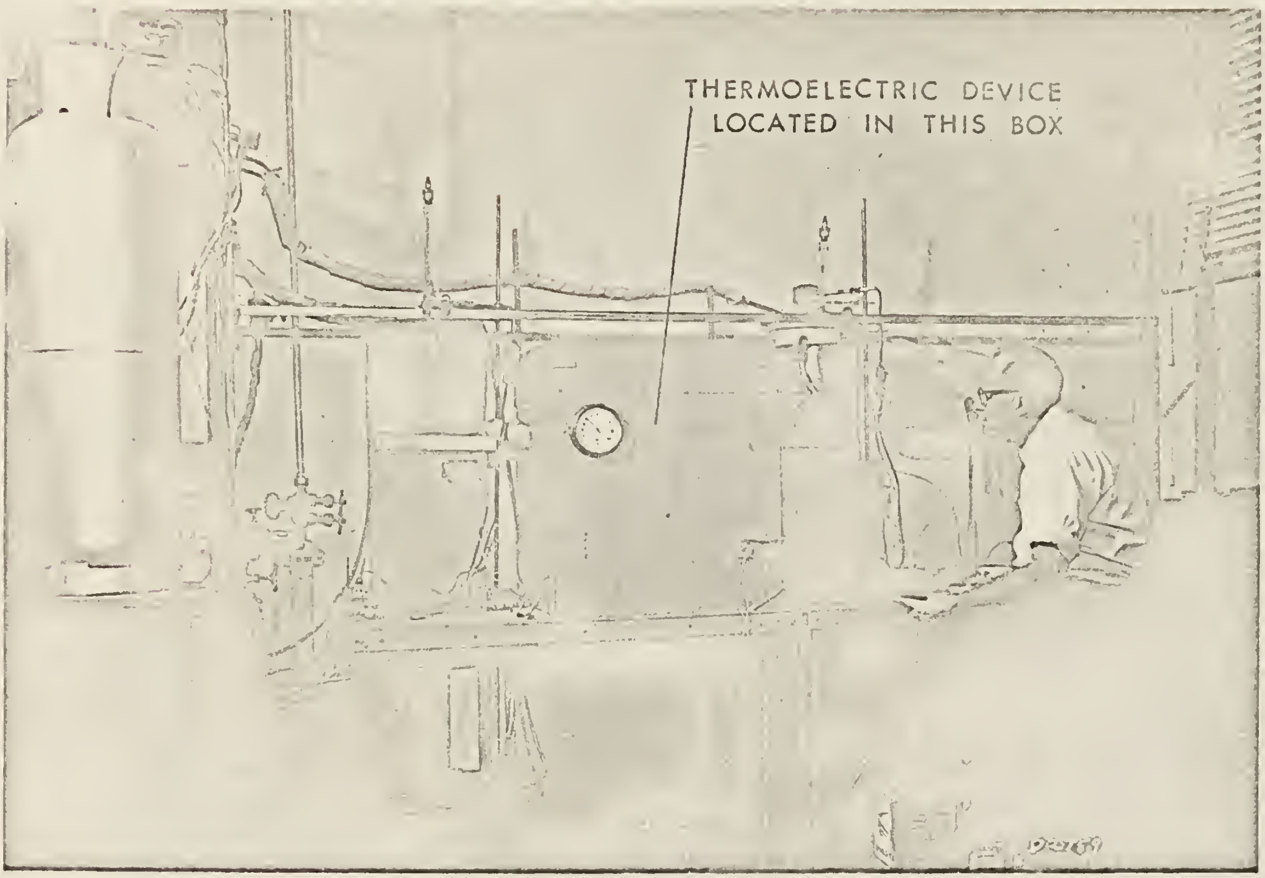
PLATE V


SUPPLY TANK
HEAT EXCHANGERS
COOLING LIQUID INLET
VACUUM PUMP
VACUUM TANK
WATER BATH WITH HEATER STIRRER AND THERMOREGULATOR
FILTER
LIQUID IN
FILTER
WATER BATH WITH HEATER STIRRER AND THERMOREGULATOR
OVERFLOW RETURN TUBES
FIBERGLAS LOOSE FILL INSULATION
T. E. DEVICE
INSULATED BOX
RETURN TANK
SOLENOID VALVES
FLOW METERS
BATCH WEIGHT SCALE
NEEDLE VALVES
LIQUID OUT
PUMP
FILTER

EXPLANATION OF PLATE VI

View of the experimental equipment in the laboratory.

PLATE VI



operate for 4 to 6 hours to "boil off" the air from the water. This deaerated water was then removed from the vacuum tank by means of a water syphon system and then pumped into the main supply tank for release into the water baths. It was found that deaerated water helped to maintain a more constant flow through the system. Air in the water can accumulate at various locations in the system particularly in the needle valves and precision flowraters thus causing variations in the water flow rate.

A return tank provided a method of collecting the liquid from the overflow tubes and the outlets from the thermoelectric device. A pump connected to the return tank allowed the liquid to be either transferred to the vacuum tank for deaerating or to the main supply tank for return to the water baths. A high quality water filter was installed after the water pump to further assure clean water for the system.

A 500-gram precision scale provide for accurate batch weighing of 300 gram samples of the liquid flowing through each circuit. Electric solonoid valves provided for selection of the liquid from each flow circuit for batch weighing. A sample was taken each time temperature and power data were noted. Any change in flow rate was thus quickly ascertained and corrected.

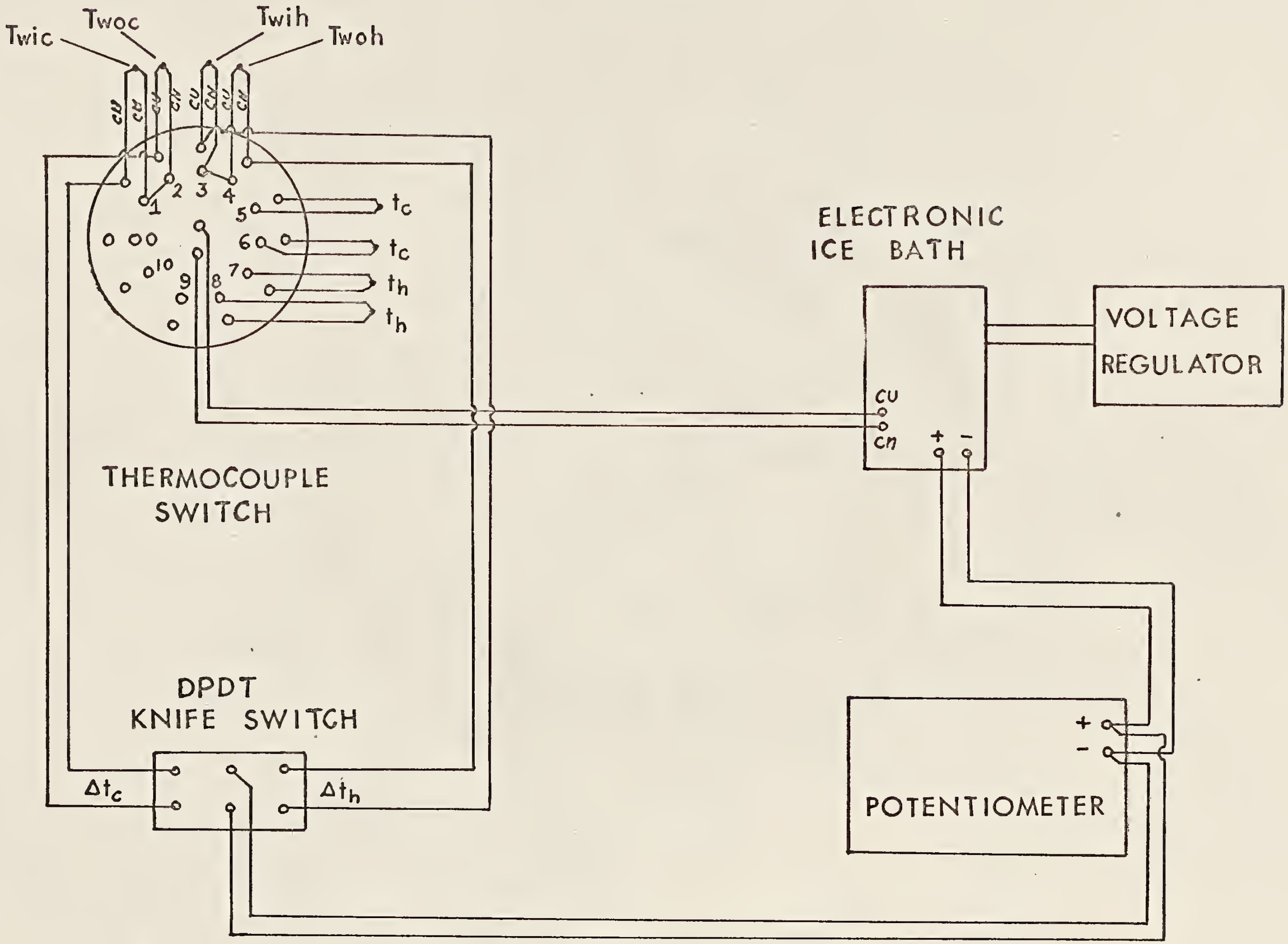
Precision flowrators were used to determine the approximate rate of flow through each circuit. However final determination of the flow rate was made with the precision scale.

A diagram of the temperature measurement instrumentation is shown in Plate VII. Temperatures were measured using calibrated copper-constantan thermocouples connected to a high quality thermocouple switch. The output terminals on this switch were connected to the input terminals of an electronic ice bath. The electronic ice bath provided a highly stable reference

EXPLANATION OF PLATE VII

Diagram of temperature measurement instrumentation.

PLATE VII


Twic
Twoc
Twih
Twoh
cu
cn
1
2
3
4
5
6
7
8
9
10
t_c
t_c
t_h
t_h
THERMOCOUPLE
SWITCH
ELECTRONIC
ICE BATH
VOLTAGE
REGULATOR
+
-
DPDT
KNIFE SWITCH
Δt_c
Δt_h
POTENTIOMETER

junction for the measurement system without the need for inaccurate water type ice baths. The output from the electronic ice bath provided a precise millivolt signal that could be accurately measured by a precision potentiometer. The supply voltage to the electronic ice bath was precisely regulated to further assure accurate reference junction output.

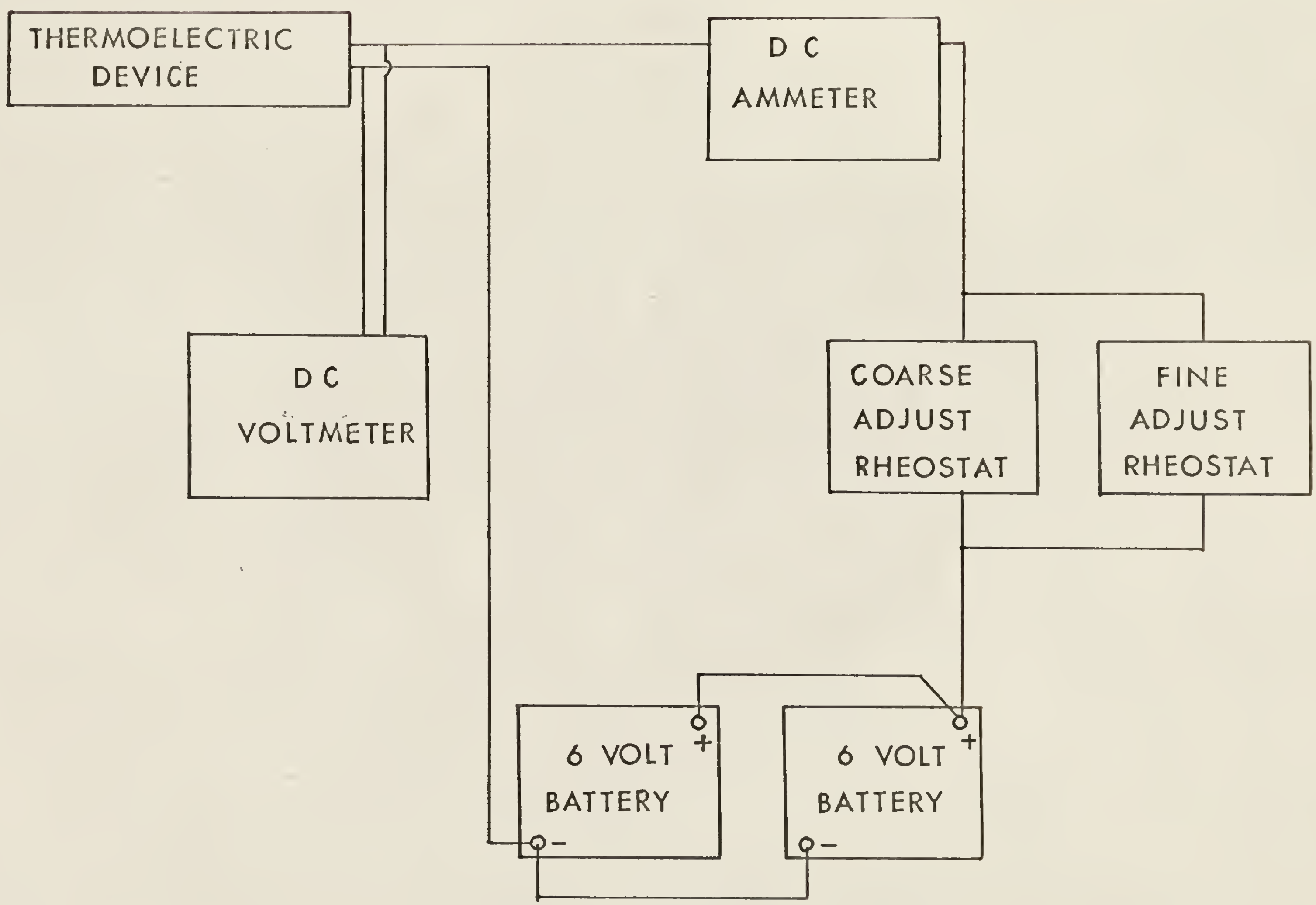
The difference between the inlet and outlet water temperatures was determined by connecting the inlet and outlet thermocouples so as to buck each other. This connection was provided at the thermocouple switch by connecting the constantan terminals together with a short length of constantan wire. The output from these thermocouples was then connected to a double pole—double throw knife switch and thence to the input terminals on the potentiometer. When it was desired to measure the temperature difference the thermocouple switch was placed in position zero (all contacts open). The knife switch could then be positioned so as to measure the temperature difference. The electronic ice bath is not connected into the measurement circuit during measurement of the temperature difference, because of the open contacts in the thermocouple switch.

Plate VIII is a diagram of the direct current power supply for the thermoelectric module and the electrical measurement instrumentation. The direct current power for the thermoelectric module was obtained from two 6-volt automotive type lead acid batteries connected in parallel. Two rheostats were connected in parallel and inserted in one of the power lines to the module. One rheostat was used to control the largest portion of the voltage drop to the module from the batteries. The second rheostat was used to drop a small portion of the voltage and acted as a fine adjustment for the current to the module. By this means the current to the module could

EXPLANATION OF PLATE VIII

Diagram of direct current power supply for thermoelectric module and electrical instrumentation.

PLATE VIII


THERMOELECTRIC DEVICE
D C AMMETER
D C VOLTMETER
COARSE ADJUST RHEOSTAT
FINE ADJUST RHEOSTAT
6 VOLT BATTERY
6 VOLT BATTERY

be precisely controlled.

A direct current ammeter and voltmeter were used to measure the power to the module. The ammeter had a full range of 15 amperes and the voltmeter a full scale range of one volt. The voltmeter could be read to .005 volts.

EXPERIMENTAL DATA

Analysis of Laboratory Data

The laboratory data were programmed for multiple-regression analysis by an IBM 1410 computer. Plates IX and X graphically show the laboratory data along with the theoretical data. Appendix C shows the data furnished the IBM 1410 computer.

The following equations were used for the statistical analysis:

1. $\pi_1 = A + B\pi_2 + C\pi_3$
2. $\pi_1 = A + B\pi_2 + C\pi_3 + D\pi_2\pi_3$
3. $\pi_1 = A + B\pi_2 + C\pi_3 + D\pi_2\pi_3 + E\pi_3^2$
4. $\pi_1 = A + B\pi_2 + C\pi_3 + D\pi_3^2$

The data were also converted to natural logarithmic values. These values were also processed by the IBM 1410 computer for the best fit analysis of an equation of the form;

$$\log \pi_1 = A + B\log \pi_2 + C\log \pi_3 \tag{82}$$

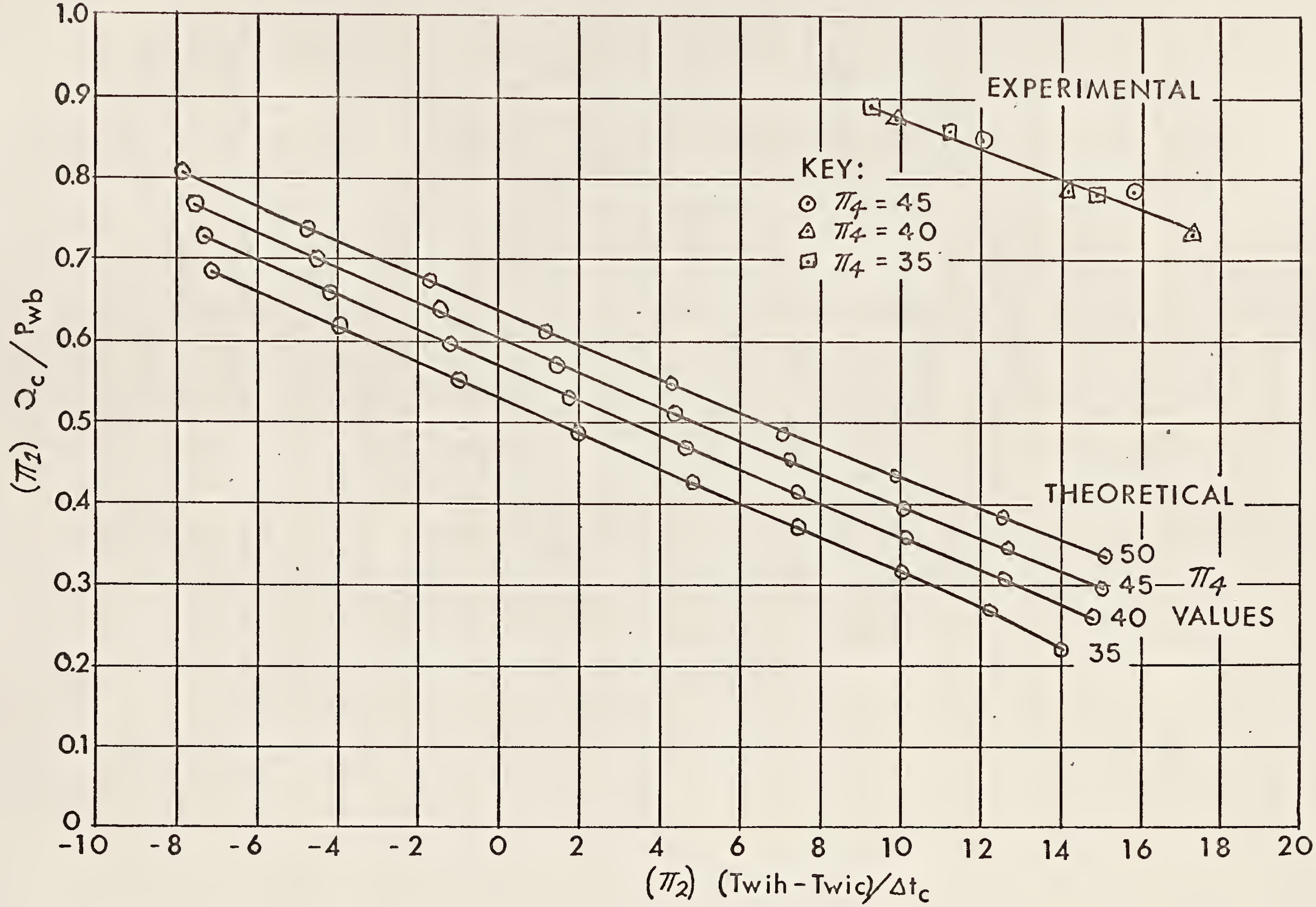
from equation (82) an equation of exponential form can be derived as follows;

$$\pi_1 = e^A (\pi_2^B)(\pi_3^C) \tag{83}$$

EXPLANATION OF PLATE IX

Plot of theoretical and laboratory data of the pi term Q_c/P_{wb} on the pi term $(T_{wih}-T_{wic})/\Delta t_c$ while holding the pi term W_c/W_h constant and the pi term $T_{wic}/\Delta t_c$ constant at selected values.

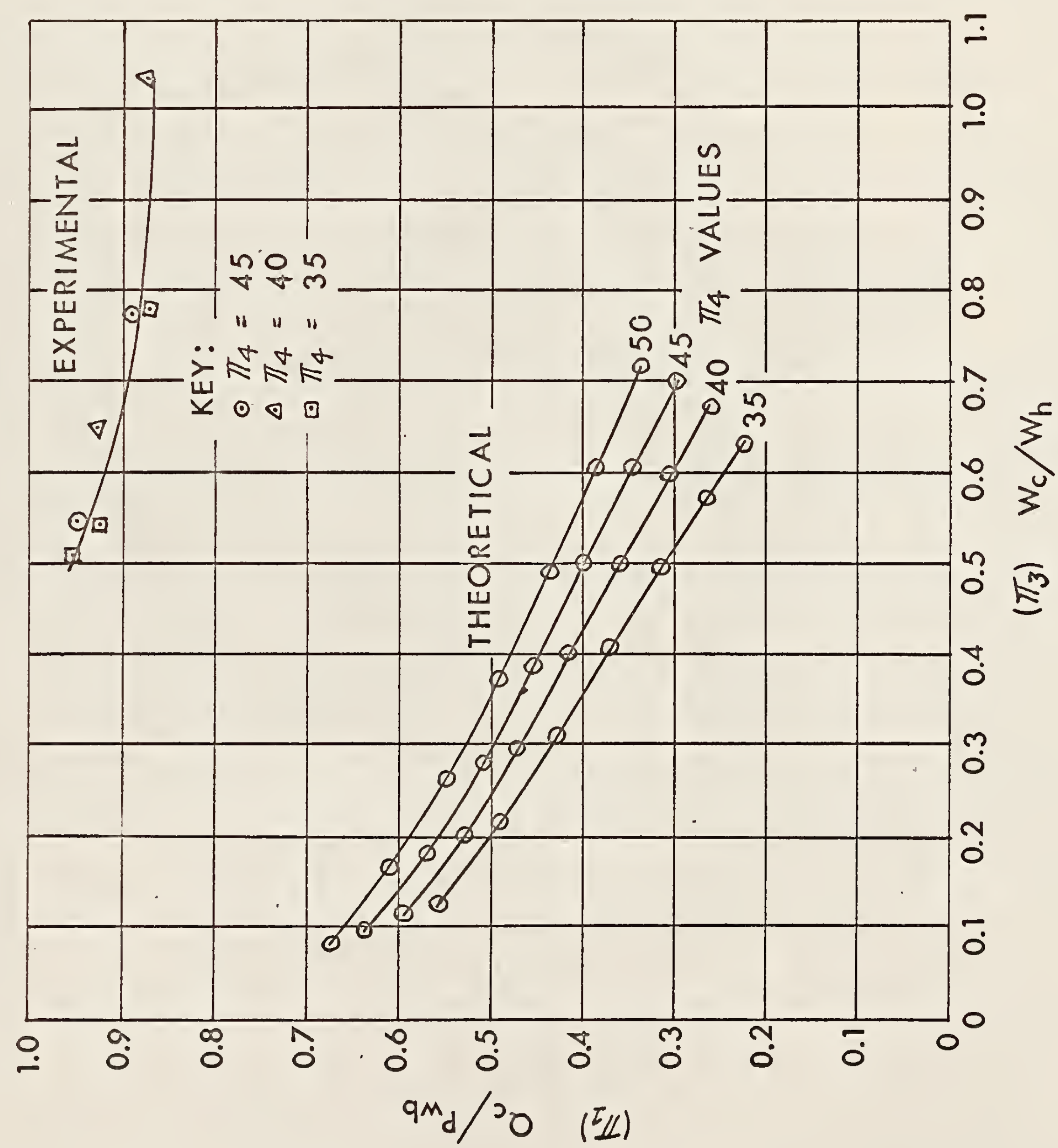
PLATE IX


EXPERIMENTAL
KEY:
π_4 = 45
π_4 = 40
π_4 = 35
THEORETICAL
50
45
40
35
π_4
VALUES
(π_1) Q_c / P_{wb}
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
16
18
20
(π_2) (Twih - Twic)/Δt_c

EXPLANATION OF PLATE X

Plot of theoretical and laboratory data of the pi term Q_c/P_{wb} on the pi term W_c/W_h while holding the pi term $(T_{wih}-T_{wic})/\Delta t_c$ constant and the pi term $T_{wic}/\Delta t_c$ constant at selected values.

PLATE X


EXPERIMENTAL
THEORETICAL
KEY:
⊙ π_4 = 45
△ π_4 = 40
⊡ π_4 = 35
50
45
40
35
π_4 VALUES
(π_3) W_c/W_h
(π_1) Q_c/P_{wb}
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1

From the multiple regression analysis by the IBM 1410 computer the following polynomial equation was determined to provide adequate precision for engineering design;

$$\pi_1 = 1.18418 - 0.02457\ \pi_2 - 0.05885\ \pi_3 \tag{84}$$

$$(R^2 = 0.859\ ,\ s_{y \cdot x} = 0.02693)$$

The exponential equation obtained from the computer was as follows;

$$\pi_1 = 2.0941\ (\pi_2)^{-0.37515}\ (\pi_3)^{-0.04366} \tag{85}$$

$$(R^2 = 0.874\ ,\ s_{y \cdot x} = 0.03015)$$

Comparison of Experimental and Theoretical Heat
Sink Heat Transfer Coefficients

From the comparison of laboratory data with theoretical data a large difference in coefficient of performance was noted. It was concluded that part of this difference could be accounted for by a difference in the heat transfer coefficient of the laboratory device as compared to a theoretical flat plate.

A comparison can be derived by using the junction temperatures as a base for calculating the log mean temperature difference between the junctions and the water being cooled or heated.

The equation used was as follows:

$$h = \frac{q}{A \Delta t_{ln}}$$

where:

h = heat transfer coefficient - Btu/hr ft^2 ^{o}F

A = area of heat transfer surface - ft^2

q = rate of heat transfer - Btu/hr

$$\Delta t_{ln} = \frac{\Delta t}{\ln \frac{t_{wi} - t_j}{t_{wo} - t_j}}$$

and:

t_{wi} = inlet water temperature - ^oF

t_{wo} = outlet water temperature - ^oF

t_j = junction temperature - ^oF

The calculated heat transfer coefficients are shown in Tables 2 and 3. Also shown are the ratio between the experimental values and the theoretical values. From this data it was concluded that the heat transfer coefficient of the experimental device was greater than the coefficient for a theoretical flat plate under the assumed conditions. The ratio of the experimental to the theoretical heat transfer coefficient related to the cold and hot junctions was about 3.32:1 and 2.46:1, respectively.

DISCUSSION

The results of this research indicate that the methods and equipment used to conduct the laboratory tests were adequate to determine dimensionless term relationships for engineering design requirements. A device having larger capacity for cooling could improve on the accuracy of laboratory measurements by providing a greater temperature drop as the liquid passes through the heat exchanger. Also the flow rates could then be increased by providing adequate systems for handling the flow of liquid and for liquid bath heating and/or cooling.

Table 2. Cooling sink heat transfer coefficient related to cold junction temperature.

Test No.	Heat Transfer Btu/Hr.	Junction Temp. ^oF	Temp. Drop ^oF	Twic ^oF	Twoc ^oF	Δt_{ln} ^oF	Heat Transfer Coefficient: Experimental Btu/Hr ^oF ft^2	Heat Transfer Coefficient: Theoretical Btu/Hr ^oF ft^2	Ratio of Coefficients Experimental to Theoretical
1.	37.8	80.9	2.0	90.0	88.0	8.06	171.8	54.5	3.15
2.	39.8	80.5	2.0	90.0	88.0	8.47	172.1	55.9	3.08
3.	36.0	71.3	2.0	79.9	77.9	7.55	174.7	51.5	3.39
4.	37.8	71.0	2.0	80.0	78.0	7.94	174.4	52.8	3.31
5.	35.2	61.8	2.0	70.0	68.0	7.15	180.3	49.3	3.66
6.	37.0	61.3	2.0	70.0	68.0	7.66	176.9	50.6	3.50
7.	37.8	61.0	2.0	70.0	68.0	7.96	173.9	51.2	3.40
8.	32.0	61.9	2.0	70.0	68.0	7.05	166.3	47.1	3.53
9.	34.0	61.4	2.0	70.0	68.0	7.56	164.7	48.5	3.40
10.	35.0	60.9	2.0	70.0	68.0	8.06	159.1	49.2	3.23
11.	31.8	72.1	2.0	80.0	78.0	6.85	170.1	48.4	3.51
12.	36.0	71.2	2.0	80.0	78.0	7.76	169.9	51.5	3.30
13.	No junction temperatures recorded.								
14.	34.2	81.6	2.0	90.0	88.0	7.35	170.4	51.8	3.29
15.	36.0	80.0	2.0	90.0	88.0	8.96	147.2	53.2	2.77
Averages							169.4	51.1	3.32

Table 3. Heating sink heat transfer coefficient related to hot junction temperature.

Test No.	Heat Transfer Btu/Hr.	Junction Temp. ^oF	Temp. Rise ^oF	Twih ^oF	Twoh ^oF	Δt_{ln} ^oF	Heat Transfer Coefficient Experimental Btu/Hr ^oF ft^2	Heat Transfer Coefficient Theoretical Btu/Hr ^oF ft^2	Ratio of Coefficients Experimental to Theoretical
1.	75.0	128.3	3.1	110.0	113.1	16.71	164.4	65.8	2.50
2.	75.2	126.3	2.1	110.0	112.1	15.23	180.9	80.1	2.26
3.	72.7	118.0	4.2	99.9	104.1	15.91	167.4	54.3	3.08
4.	74.9	115.6	2.6	100.0	102.6	14.26	192.4	69.8	2.76
5.	71.7	107.3	3.2	90.0	93.2	15.69	167.4	59.8	2.80
6.	73.9	105.5	2.2	90.0	92.2	14.38	188.2	73.0	2.58
7.	72.8	105.3	2.0	90.0	92.0	12.89	206.9	76.4	2.70
8.	67.2	115.2	2.1	100.0	102.1	14.12	174.3	73.5	2.37
9.	71.2	107.9	2.1	92.6	94.7	14.22	183.4	74.0	2.48
10.	70.2	107.3	2.0	88.8	90.8	17.48	147.1	74.4	1.98
11.	69.7	131.6	2.2	114.8	117.0	15.67	162.9	76.3	2.14
12.	72.2	115.0	2.0	100.0	102.0	13.97	189.3	78.0	2.42
13.	No junction temperatures recorded.								
14.	73.9	138.5	2.2	121.7	123.9	15.67	172.7	80.1	2.16
15.	75.8	130.1	2.1	114.1	116.2	14.92	186.1	81.3	2.29
Averages							177.4	72.6	2.46

Multiple regression was used to determine the relationships between the pi terms involved in the analysis. Polynomial equations of the form $\pi_1 = A + B\pi_2 + C\pi_3$ and exponential equations of the form $\pi_1 = C\pi_2{}^a\pi_3{}^b$ were developed from computer programs. The equation showing adequate fit to the data and having the least number of terms was chosen from the group of equations solved by the computer programs.

As shown in Plate IX the relationship of π_1(Qc/Pwb) to π_2 (Twih-Twic/Δtc) was almost linear with π_3(Wc/Wh) held constant. Plate X shows a non-linear relationship of π_1(Qc/Pwb) to π_3(Wc/Wh) with π_2(Twih-Twic/Δtc) held constant. The above relationships were approximately true for both theoretical and experimental data.

The mercury thermoregulators used to maintain constant water bath temperatures were very sensitive. Difficulty was experienced when mounting these regulators directly on the sides of the water baths. The slightest vibration from the stirrer motors could cause the regulator relay to chatter. The final solution to this problem was to mount the regulators on a rod which was isolated from the water baths.

CONCLUSIONS

The following conclusions were drawn from the analysis and data presented:

1. The method of dimensional analysis can be used to determine the operating characteristics of a thermoelectric heating/cooling device. The device can be either operated with liquid or air flowing through the heat sinks or liquid through one sink and air through the other. For example the device could be used to cool and heat water or cool water and heat air.

2. The slopes of the graphs for the experimental and theoretical data were not exactly the same. It is believed that experimental error can account for part of this inaccuracy. The experimental data showed that for $\pi_2(\frac{Twih - Twic}{\Delta tc})$ constant, $\pi_3(\frac{Wc}{Wh})$ decreased in value as π_1 (cooling C.O.P.) increased in value and appeared to be a curvilinear function. Also when $\pi_3(\frac{Wc}{Wh})$ was held constant, $\pi_2(\frac{Twih - Twic}{\Delta tc})$ decreased as π_1 (cooling C.O.P.) increased and the relationship was essentially linear. Both relationships were in general agreement with the predicted functions as derived from the theoretical analysis.

3. Another factor which could have caused a deviation between experimental and theoretical results could be a difference in the heat transfer coefficient of the experimental device. The theoretical calculations were based on an emperical equation for a flat plate under laminar flow conditions. Although the experimental device approached a flat plate configuration, the liquids entered and left the heat transfer system at right angles. This could have caused entrance and exit effects that the theoretical analysis did not include.

4. The experimental data did not show any measurable difference due to different water inlet temperatures at the cooling side of the device. From the theoretical data an error of less than 0.1 degree Fahrenheit in the measurement of the temperature drop of the water being cooled could have nullified the theoretical difference due to the inlet water temperature. Since the inlet water temperature appeared to have no significant effect, the fourth Pi term $(\frac{Twic}{t_c})$ was eliminated from the statistical analysis of the experimental data.

5. An empirical equation for the type of heat sinks used in this

research was not available. Therefore close agreement between actual values of experimental and theoretical data could not be expected.

SUGGESTIONS FOR FURTHER RESEARCH

Laboratory experiments of the type described here, require highly accurate instruments. Instruments for measuring the electrical power delivered to the device should be as accurate as possible. Thermocouples for determining the temperature drop of the liquid being cooled should be calibrated.

It was possible with the instruments available and calibrated thermocouples to measure a temperature change of $2^{\circ}F$ in the liquid being cooled. In future research a larger device should be assembled so as to provide a greater heat transfer capacity. This greater heat transfer capacity would result in a greater temperature drop of the liquid being cooled. The result should be a greater degree of precision in the measurement of the temperature drop of the liquid.

Other investigators who may perform similar research should carefully consider the precise control of liquid flow, and assure themselves of reliable equipment. A closed system for the liquid circulation with suitable filters should be considered for future research. However the open type system and filters used in these tests proved to be satisfactory.

Future research should be carried out with other temperature drops of the liquid being cooled. The tests reported were performed for the liquid being cooled $2^{\circ}F$. Additional research would provide a family of equations related to various temperature drops. Such equations could then be applied to engineering design for a particular device.

Also these tests were performed with a current flow of 15 amperes. Future tests should be conducted on a device with other currents, i.e. 20, 25, and 30 amperes, or up to the limit of current for the particular thermoelectric module being used.

Future research should encompass the preliminary testing of a particular device to determine the heat transfer coefficient of the heat sinks, if a satisfactory theoretical heat transfer coefficient is not available.

ACKNOWLEDGMENTS

The author owes much to his wife Jean and sons Jim and Bob, for their patience, cooperation and loving support for his work. They must have felt at times, that they did not have a husband or a father.

Many thanks are expressed to Dr. Truman E. Hienton, Chief, Farm Electrification Research Branch, Agricultural Research Service, U.S. Department of Agriculture, for his wisdom and patience. Largely due to his persuasion the author accepted the challenge and opportunity to do advanced work. His kindness and consideration are much appreciated.

Grateful acknowledgment is expressed for my advisors who contributed with their support, guidance and thoughtfulness. Thanks to Dr. George H. Larson, Head, Agricultural Engineering Department and Dr. Teddy O. Hodges and Professor Ralph I. Lipper of the Agricultural Engineering Department. The author appreciates very much the inspiration and guidance rendered to him by Professor Wilson Tripp, Mechanical Engineering Department.

The author also wishes to thank Herbert D. Ball, Mechanical Engineering Department for his interest and helpful advice, Dr. Arlin H. Feyerherm, Department of Statistics, for his guidance, Professor Michael H. Miller, Mathematics Department and Computing Center, for his guidance in programming the IBM 1410 computer, Dr. Jacob J. Smaltz, Industrial Engineering Department, for instruction and guidance in the use of the IBM 1620 computer.

The understanding, helpfulness and moral support received from the late Chester Paul Davis, Jr., are also acknowledged.

The high quality of the thermoelectric modules and accessories supplied by Frigistors Ltd., Montreal, Canada are appreciated. Also the author acknowledges the use of Frigistor module performance characteristics as

part of the theoretical analysis.

And to the friends who were concerned and contributed with their moral support, the author expresses deep appreciation and thanks.

And last but not least, to my mother the author gives many thanks for all her sacrifices. She advised the author to accept this opportunity when she knew that it would separate our family at a time when she had hoped we would be near to them.

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APPENDIX A

Determination of Conversion Factor to Convert Thermal Conductivity from Metric Units to British Thermal Units

Determination of conversion factor to convert from gram calories per square centimeter per degree Centigrade per second/centimeter to British Thermal Units per square foot per degree Fahrenheit per hour/foot.

Gram calories x 3.968×10^{-3} = British Thermal Units

Degrees Centigrade x 1.8 = degrees Fahrenheit

Square centimeters x 1.076×10^{-3} = square feet

$$\frac{\text{Centimeters}}{30.48} = \text{feet}$$

$$\frac{\text{Seconds}}{3600} = \text{hours}$$

and;

$$k = \frac{\text{gm. cal.}}{\text{sec. cm.}^2\ {}^{\circ}\text{C./cm.}}$$

Substitution of these conversion units in the above equation results in the following relationship;

$$k = \frac{(\text{gm. cal.})(3.9681)(10^{-3})}{({}^{\circ}\text{C.})(1.8)(\text{cm.}^2)(1.076)(10^{-3})\frac{(\text{sec.})}{3600}/\frac{(\text{cm.})}{30.48}}$$

$$k = \frac{(3.968)(3600)}{(1.8)(1.076)(30.48)}$$

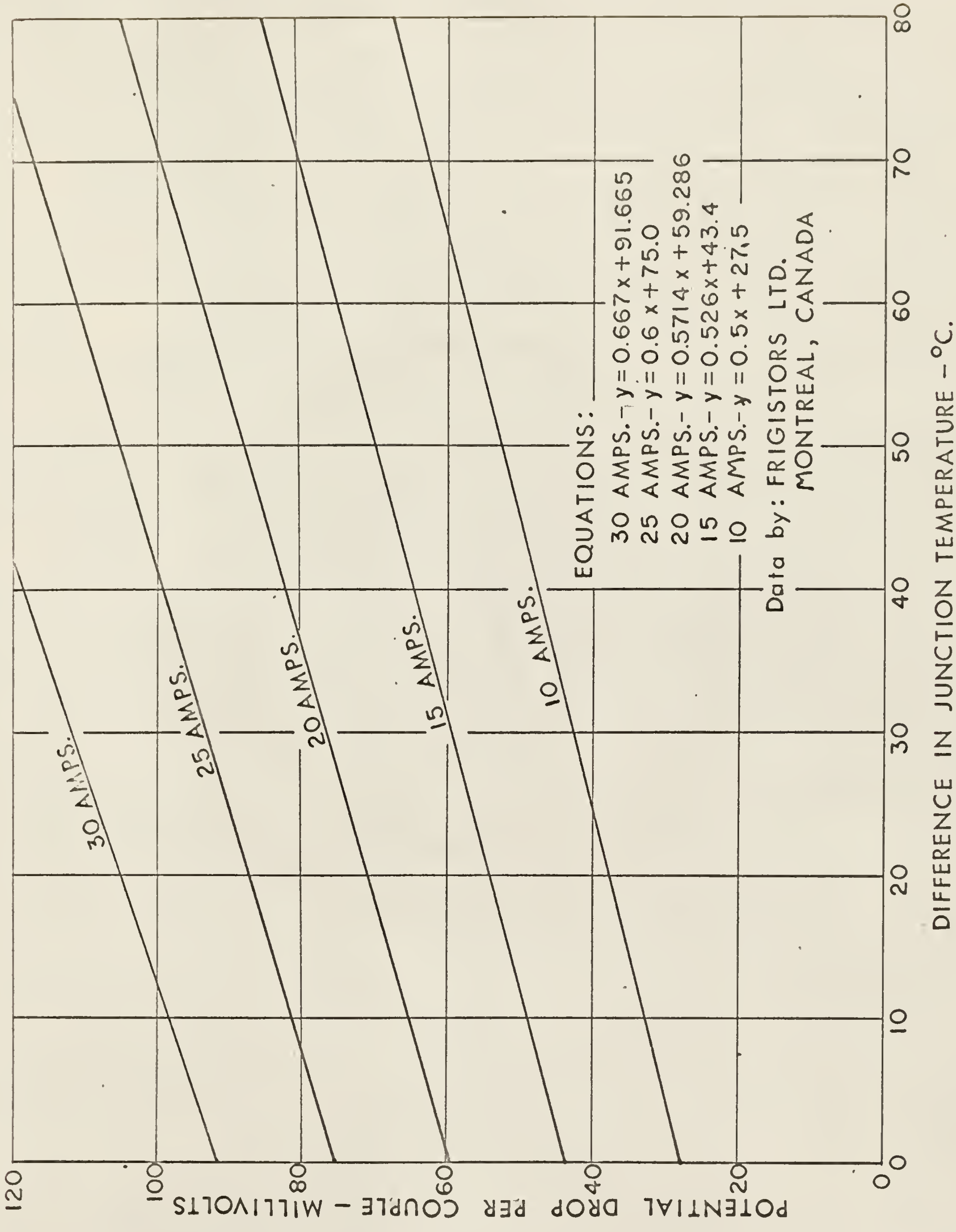
$$k = 241.97\ \frac{\text{Btu.}}{\text{hr. ft.}^2\ {}^{\circ}\text{F./ft.}}$$

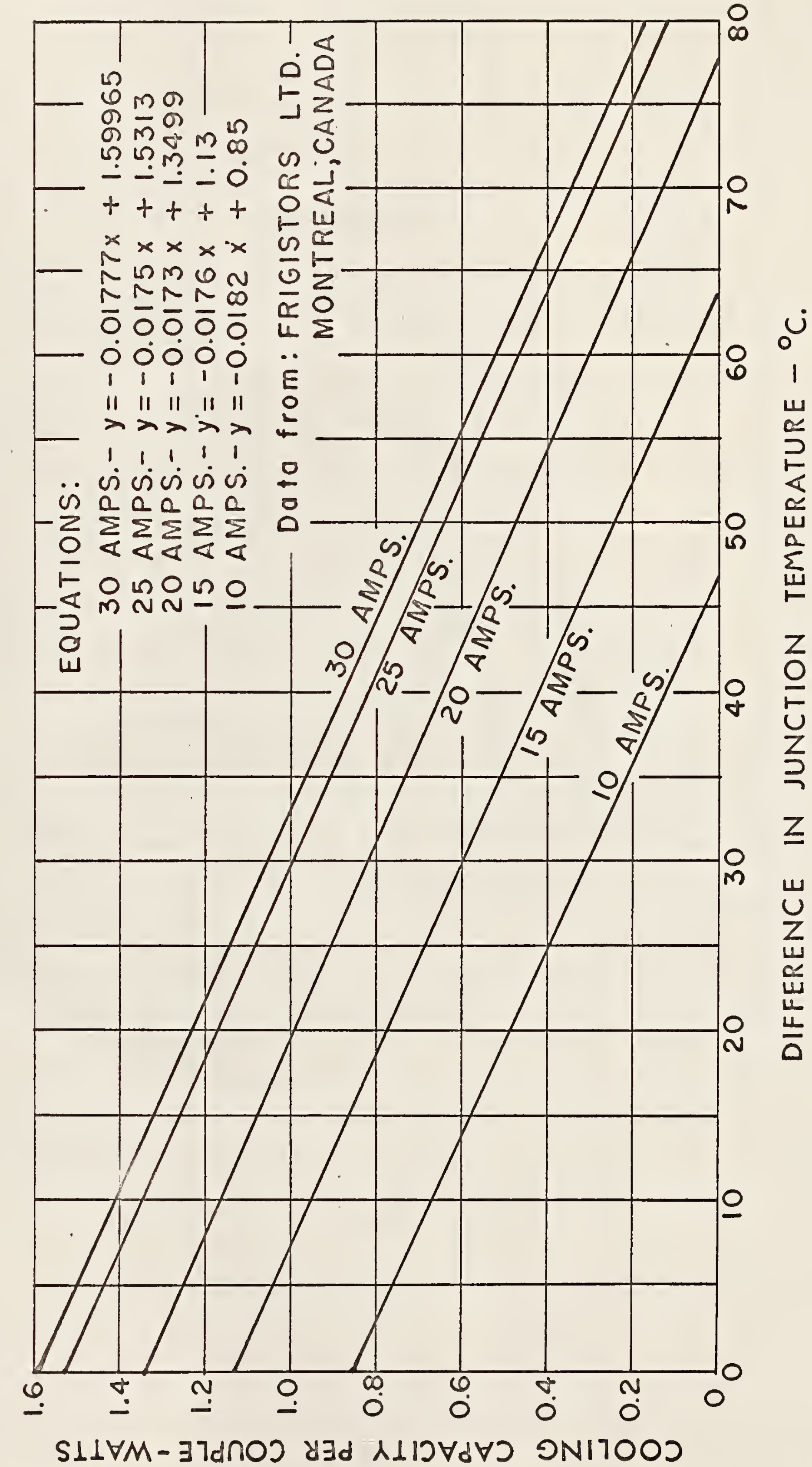
Therefore:

$$\frac{\text{Btu.}}{\text{hr. ft.}^2\ {}^{\circ}\text{F./ft.}} = \frac{\text{gm. cal.}}{\text{sec. cm.}^2\ {}^{\circ}\text{C./cm.}}\ (241.97)$$

APPENDIX B

Sintered Frigistor Performance Data


POTENTIAL DROP PER COUPLE – MILLIVOLTS
DIFFERENCE IN JUNCTION TEMPERATURE – °C.
0
20
40
60
80
100
120
0
10
20
30
40
50
60
70
80
30 AMPS.
25 AMPS.
20 AMPS.
15 AMPS.
10 AMPS.
EQUATIONS:
30 AMPS.– y = 0.667 x + 91.665
25 AMPS.– y = 0.6 x + 75.0
20 AMPS.– y = 0.5714 x + 59.286
15 AMPS.– y = 0.526x + 43.4
10 AMPS.– y = 0.5x + 27.5
Data by: FRIGISTORS LTD.
MONTREAL, CANADA


EQUATIONS:
30 AMPS.- y = -0.01777x + 1.59965
25 AMPS.- y = -0.0175 x + 1.5313
20 AMPS.- y = -0.0173 x + 1.3499
15 AMPS.- y = -0.0176 x + 1.13
10 AMPS.- y = -0.0182 x + 0.85
Data from: FRIGISTORS LTD.
MONTREAL, CANADA
30 AMPS.
25 AMPS.
20 AMPS.
15 AMPS.
10 AMPS.
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
60
70
80
COOLING CAPACITY PER COUPLE - WATTS
DIFFERENCE IN JUNCTION TEMPERATURE - °C.

APPENDIX C

Data Programmed for Multiple-regression Analysis by IBM 1410 Computer

Table 3. Condensed theoretical data and program data for multiple regression analysis by IBM 1410 computer.

THEORETICAL POLYNOMIAL					
π_1	π_2	π_3	π_4	$\pi_2 \pi_3 \pi_4$	π_4^2
.2152	10.0	.6329	35.0	221.5150	1225.0
.2646	10.0	.5797	35.0	202.8950	1225.0
.3164	10.0	.5036	35.0	176.260	1225.0
.3707	10.0	.4113	35.0	143.9550	1225.0
.4278	10.0	.3141	35.0	109.9350	1225.0
.4877	10.0	.220	35.0	77.0	1225.0
.5508	10.0	.1355	35.0	47.4250	1225.0
.2554	10.0	.6780	40.0	271.20	1600.0
.3051	10.0	.6009	40.0	240.360	1600.0
.3571	10.0	.5079	40.0	203.160	1600.0
.4116	10.0	.4045	40.0	161.80	1600.0
.4688	10.0	.3012	40.0	120.480	1600.0
.5288	10.0	.2043	40.0	81.720	1600.0
.5919	10.0	.1201	40.0	48.040	1600.0
.2946	10.0	.7071	45.0	318.1950	2025.0
.3445	10.0	.6107	45.0	274.8150	2025.0
.3967	10.0	.5056	45.0	227.520	2025.0
.4513	10.0	.3945	45.0	177.5250	2025.0
.5086	10.0	.2861	45.0	128.7450	2025.0
.5686	10.0	.1876	45.0	84.3750	2025.0
.6316	10.0	.1048	45.0	47.160	2025.0
.3328	10.0	.7240	50.0	362.0	2500.0
.3829	10.0	.6143	50.0	307.650	2500.0
.4352	10.0	.4975	50.0	248.750	2500.0
.4899	10.0	.3800	50.0	190.0	2500.0
.5472	10.0	.2688	50.0	134.40	2500.0
.6072	10.0	.1703	50.0	85.150	2500.0
.6701	10.0	.0898	50.0	44.90	2500.0

Table 3 (cont.).

THEORETICAL POLYNOMIAL					
π_1	π_2	π_3	π_4	$\pi_2 \pi_3 \pi_4$	π_4^2
.2152	14.1042	.50	35.0	246.8235	1225.0
.2646	12.3314	.50	35.0	215.7995	1225.0
.3164	10.1093	.50	35.0	176.9128	1225.0
.3707	7.5992	.50	35.0	132.9860	1225.0
.4278	4.8803	.50	35.0	85.4053	1225.0
.4877	2=*527	.50	35.0	35.9223	1225.0
.5508	-0.8905	.50	35.0	- 15.5838	1225.0
.6172	- 3.9311	.50	35.0	-68.7943	1225.0
.6873	-7.0192	.50	35.0	-122.8360	1225.0
.2554	14.8612	.50	40.0	297.2240	1600.0
.3051	12.6667	.50	40.0	253.3340	1600.0
.3571	10.2022	.50	40.0	204.0440	1600.0
.4116	7.5539	.50	40.0	151.0780	1600.0
.4688	4.7470	.50	40.0	94.940	1600.0
.5288	1.8495	.50	40.0	36.990	1600.0
.5919	- 1.1205	.50	40.0	- 22.410	1600.0
.6582	- 4.1630	.50	40.0	- 83.260	1600.0
.7281	-7.2599	.50	40.0	-145.1980	1600.0
.2946	15.1715	.50	45.0	341.3588	2025.0
.3445	12.7526	.50	45.0	286.9335	2025.0
.3967	10.1319	.50	45.0	227.9678	2025.0
.4513	7.3707	.50	45.0	165.8408	2025.0
.5086	4.5188	.50	45.0	101.6730	2025.0
.5686	1.5944	.50	45.0	35.8740	2025.0
.6316	- 1.4093	.50	45.0	- 31.7093	2025.0
.6978	- 4.4674	.50	45.0	-100.5165	2025.0
.7674	- 7.5479	.50	45.0	-169.8278	2025.0
.3328	15.2394	.50	50.0	380.9850	2500.0
.3829	12.6574	.50	50.0	316.4350	2500.0
.4352	9.9418	.50	50.0	248.5450	2500.0
.4899	7.1355	.50	50.0	178.3875	2500.0
.5472	4.2249	.50	50.0	105.6225	2500.0
.6072	1.2669	.50	50.0	31.6725	2500.0
.6701	- 1.7206	.50	50.0	- 43.0150	2500.0
.7361	- 4.7692	.50	50.0	-119.230	2500.0
.8055	- 7.8473	.50	50.0	-196.1825	2500.0

Table 3 (cont.).

THEORETICAL LOGARITHMIC			
π_1	π_2	π_3	π_4
-1.5362	2=9957	- .4574	3.5553
-1.3295	2.9957	- .5452	3.5553
-1.1507	2.9957	- .6860	3.5553
- .9924	2.9957	- .8884	3.5553
- .8491	2.9957	-1.1580	3.5553
- .7181	2.9957	-1.5141	3.5553
- .5964	2.9957	-1.9988	3.5553
-1.3649	2.9957	- .3886	3.6889
-1.1871	2.9957	- .5093	3.6889
-1.0297	2.9957	- .6775	3.6889
- .8877	2.9957	- .9051	3.6889
- .7576	2.9957	-1.1999	3.6889
- .6371	2.9957	-1.5882	3.6889
- .5244	2.9957	-2.1194	3.6889
-1.2221	2.9957	- .3466	3.8067
-1.0657	2.9957	- .4931	3.8067
- .9246	2.9957	- .6820	3.8067
- .7956	2.9957	- .9301	3.8067
- .6761	2.9957	-1.2514	3.8067
- .5646	2.9957	-1.6734	3.8067
- .4595	2.9957	-2.2557	3.8067
-1.1002	2.9957	- .3230	3.9120
- .9599	2.9957	- .4873	3.9120
- .8319	2.9957	- .6982	3.9120
- .7136	2.9957	- .9676	3.9120
- .6029	2.9957	-1.3138	3.9120
- .4989	2.9957	-1.7702	3.9120
- .4003	2.9957	-2.4102	3.9120

Table 3 (concl.).

THEORETICAL LOGARITHMIC			
π_1	π_2	π_3	π_4
-1.5362	3=1824	- .6931	3.5553
-1.3295	3.1060	- .6931	3.5553
-1.1507	3.0012	- .6931	3.5553
- .9924	2.8679	- .6931	3.5553
- .8491	2.70	- .6931	3.5553
- .7181	2.4893	- .6931	3.5553
- .5964	2.2093	- .6931	3.5553
- .4826	1.8032	- .6931	3.5553
- .3750	1.0922	- .6931	3.5553
-1.3649	3.2133	- .6931	3.6889
-1.1871	3.1209	- .6931	3.6889
-1.0297	3.0058	- .6931	3.6889
- .8877	2.8653	- .6931	3.6889
- .7576	2.6910	- .6931	3.6889
- .6371	2.4723	- .6931	3.6889
- .5244	2.1837	- .6931	3.6889
- .4182	1.7642	- .6931	3.6889
- .3173	1.0080	- .6931	3.6889
-1.2221	3.2257	- .6931	3.8067
-1.0657	3.1247	- .6931	3.8067
- .9246	3.0023	- .6931	3.8067
- .7956	2.8548	- .6931	3.8067
- .6761	2.6754	- .6931	3.8067
- .5646	2.4505	- .6931	3.8067
- .4595	2.1507	- .6931	3.8067
- .3598	1.7107	- .6931	3.8067
- .2647	.8969	- .6931	3.8067
-1.1002	3.2284	- .6931	3.9120
- .9600	2.5382	- .6931	3.9120
- .8319	2.9928	- .6931	3.9120
- .7136	2.8412	- .6931	3.9120
- .6029	2.6550	- .6931	3.9120
- .4989	2.4219	- .6931	3.9120
- .4003	2.1138	- .6931	3.9120
- .3064	1.6546	- .6931	3.9120
- .2163	.7667	- .6931	3.9120

APPENDIX D

Condensed Laboratory Test Data

Table 4. Condensed laboratory data and program data for multiple regression by IBM 1410 computer.

		LABORATORY POLYNOMIAL		
π_1	π_2	π_3	$\pi_2 \pi_3$	π_3^2
.9520	10.0	.5190	5.190	.2694
.9430	10.0	.5190	5.560	.3091
.9290	10.0	.6560	6.560	.4303
.9270	10.0	.5510	5.510	.3036
.8890	10.0	.7810	7.810	.6099
.8760	10.0	.7860	7.860	.6178
.8740	10.0	1.040	10.40	1.0816
.8880	9.40	.4990	4.690	.2490
.8740	10.0	.4990	4.990	.2490
.8560	11.30	.5010	5.6613	.2510
.8470	12.10	.4990	6.0379	.2490
.7810	14.30	.50	7.150	.250
.7810	15.90	.5090	8.0931	.2591
.780	15.0	.50	7.50	.250
.7340	17.40	.5020	8.7348	.2520
		LABORATORY LOGARITHMIC		
-.0492	2.3026	-.6559		
-.0587	2.3026	-.5870		
-.0736	2.3026	-.4216		
-.0758	2.3026	-.5960		
-.1177	2.3026	-.2472		
-.1324	2.3026	-.2408		
-.1347	2.3026	-.0392		
-.1188	2.2407	-.6951		
-.1347	2.3026	-.6951		
-.1555	2.4248	-.6911		
-.1661	2.4932	-.6951		
-.2472	2.6603	-.6931		
-.2472	2.7663	-.6753		
-.2485	2.7081	-.6931		
-.3092	2.8565	-.6892		

APPENDIX E

IBM 1620 Computer FORGO Programs

Used For Theoretical Dimensional Analysis

FORGO PROGRAM FOR IBM 1620 COMPUTER
TO SOLVE FOR PI3 HOLDING PI2 CONSTANT

```
C   C        MOWRY
      TWIC=80.
      DELTC=2.
      P4=TWIC/DELTC
      P2=10.
      A=.0273
      TWIH=(P2)*(DELTC)+TWIC
      DC=15.
      WC=8.
    3 QC=WC*DELTC
      QCT=(QC)/((12.)*(3.413))
      TWOC=TWIC-DELTC
      TPC=TWOC-.1
      R1=(WC)**.4997
    9 R2=(TWIC-TPC)/(TWOC-TPC)
      R3=LOG(R2)
      DTLNC=(DELTC)/(R3)
      H1=(QC)/(A*DTLNC)
      Y1=((.0383)*(TPC+DTLNC)+9.1325)*(R1)
   19 IF(SENSE SWITCH 1)20,21
   20 PRINT 102,TPC,H1,Y1
  102 FORMAT(3F10.4)
   21 CONTINUE
      IF((ABS(H1-Y1))-50.)70,70,71
   71 TPC=TPC-1.
      GO TO 9
   70 CONTINUE
      IF((ABS(H1-Y1))-20.)60,60,61
   61 TPC=TPC-.5
      GO TO 9
   60 IF((ABS(H1-Y1))-3.)62,62,63
   63 TPC=TPC-.1
      GO TO 9
   62 IF((ABS(H1-Y1))-1.)64,64,65
   65 TPC=TPC-.-5
      GO TO 9
   64 IF((ABS(H1-Y1))-.5)67,67,68
   68 TPC=TPC-.01
      GO TO 9
   67 CONTINUE
      DELCG=(QC)/(17.46)
      FTC=TPC-DELCG
      CTC=((5.)/(9.))*(FTC-32.)
      DHC=(1.13+(.00678*CTC)-QCT)/(.0176)
      CTH=DHC+CTC
      VM=.526*DHC+43.4
```

```
      PW=((VM+.017*DC*CTC)*(12.*DC))/(1000.)
      PWB=(PW)*(3.413)
      FTH=((9.)/(5.))*(CTH)+32.
      QH=QC+PWB
      DELHG=(QH)/(17.46)
      TPH=FTH-DELHG
      WH=.1
    5 R4=(WH)**.4997
      DELTH=(QH)/(WH)
      TWOH=TWIH+DELTH
      IF(TWOH-TPH)6,4,4
    4 WH=WH+1.
      GO TO 5
    6 CONTINUE
      R5=(TWIH-TPH)/(TWOH-TPH)
      R6=LOG(R5)
      H2=((WH)*(R6))/(.0273)
      Y2=((.0383)*(TPH-(DELTH)/(R6))+9.1325)*R4
   18 IF(SENSE SWITCH 2)15,10
   15 PRINT 101,WH,H2,Y2
  101 FORMAT(3F10.4)
   10 CONTINUE
      IF((ABS(H2-Y2))-50.)52,52,53
   53 WH=WH+2.
      GO TO 5
   52 IF((ABS(H2-Y2))-20.)22,22,23
   23 WH=WH+1.
      GO TO 5
   22 IF((ABS(H2-Y2))-3.)27,27,29
   29 WH=WH+.5
      GO TO 5
   27 IF((ABS(H2-Y2))-1.)16,16,17
   17 WH=WH+.1
      GO TO 5
   16 IF((ABS(H2-Y2))-.5)24,24,25
   25 WH=WH+.05
      GO TO 5
   24 CONTINUE
      P1=(QC)/(PWB)
      P2=(TWIH-TWIC)/(DELTC)
      P3=(WC)/(WH)
      PUNCH 28,DHC,FTC,PW,DELTC,DELTH
      PUNCH 28,P1,P2,P3,TWIC,TWIH
      PUNCH 28,WH,WC,QC,QH,PWB
      PUNCH 28,TPC,TPH,FTH,P4,DC
   28 FORMAT(5F10.4)
      WC=WC+1.
      GO TO 3
      END
```

FORGO PROGRAM FOR IBM 1620 COMPUTER
TO SOLVE FOR PI2 HOLDING PI3 CONSTANT

```
C   C        MOWRY
      TWIC=80.
      DELTC=2.
      P4=TWIC/DELTC
      A=.0273
      DC=15.
      P3=.5
      WC=8.
    3 QC=WC*DELTC
      QCT=(QC)/((12.)*(3.413))
      TWOC=TWIC-DELTC
      TPC=TWOC-.1
      R1=(WC)**.4997
    9 R2=(TWIC-TPC)/(TWOC-TPC)
      R3=LOG(R2)
      DTLNC=(DELTC)/(R3)
      H1=(QC)/(A*DTLNC)
      Y1=((.0383)*(TPC+DTLNC)+9.1325)*(R1)
   19 IF(SENSE SWITCH 1)20,21
   20 PRINT 102,TPC,H1,Y1
  102 FORMAT(3F10.4)
   21 CONTINUE
      IF((ABS(H1-Y1))-50.)70,70,71
   71 TPC=TPC-1.
      GO TO 9
   70 CONTINUE
      IF((ABS(H1-Y1))-20.)60,60,61
   61 TPC=TPC-.5
      GO TO 9
   60 IF((ABS(H1-Y1))-3.)62,62,63
   63 TPC=TPC-.1
      GO TO 9
   62 IF((ABS(H1-Y1))-1.)64,64,65
   65 TPC=TPC-.05
      GO TO 9
   64 IF((ABS(H1-Y1))-.5)66,66,67
   67 TPC=TPC-.01
      GO TO 9
   66 CONTINUE
      DELCG=(QC)/(17.46)
      FTC=TPC-DELCG
      CTC=((5.)/(9.))*(FTC-32.)
      DHC=(1.13+(.00678*CTC)-QCT)/(.0176)
      CTH=DHC+CTC
      VM=.526*DHC+43.4
```

```
      PW=((VM+.017*DC*CTC)*(12.*DC))/(1000.)
      PWB=(PW)*(3.413)
      FTH=((9.)/(5.))*(CTH)+32.
      QH=QC+PWB
      DELHG=(QH)/(17.46)
      TPH=FTH-DELHG
      WH=(WC)/(P3)
      R4=(WH)**.4997
      DELTH=(QH)/(WH)
      TWIH=TPH
    5 TWOH=TWIH+DELTH
      IF(TWOH-TPH)6,4,4
    4 TWIH=TWIH-1.
      GO TO 5
    6 CONTINUE
      R5=(TWIH-TPH)/(TWOH-TPH)
      R6=LOG(R5)
      H2=((WH)*(R6))/(.0273)
      Y2=((.0383)*(TPH-(DELTH)/(R6))+9.1325)*R4
      IF(SENSE SWITCH 2)15,10
   15 PRINT 101,TWIH,H2,Y2
  101 FORMAT(3F10.4)
   10 CONTINUE
      IF((ABS(H2-Y2))-20.)52,52,53
   53 TWIH=TWIH-1.
      GO TO 5
   52 IF((ABS(H2-Y2))-3.)22,22,23
   23 TWIH=TWIH-.5
      GO TO 5
   22 IF((ABS(H2-Y2))-1.)27,27,29
   29 TWIH=TWIH-.1
      GO TO 5
   27 IF((ABS(H2-Y2))-.5)24,24,25
   25 TWIH=TWIH-.05
      GO TO 5
   24 CONTINUE
      P1=(QC)/(PWB)
      P2=(TWIH-TWIC)/(DELTC)
      PUNCH 28,DHC,FTC,PW,DELTC,DELTH
      PUNCH 28,P1,P2,P3,TWIC,TWIH
      PUNCH 28,WH,WC,QC,QH,PWB
      PUNCH 28,TPC,TPH,FTH,P4,DC
   28 FORMAT(5F10.4)
      WC=WC+1.
      GO TO 3
      END
```

APPENDIX F

Important Thermoelectric Equations

One of the most outstanding thermoelectric cooling effects is the Peltier effect. The Peltier effect is noted when two dissimilar electrical conducting materials are joined and a direct current is passed through them. The Peltier effect is noted by the fact that a temperature difference is generated between the two junctions. The defining function that describes this operation is:

$$Q_c = (\alpha_p - \alpha_h)\, T_c I - \tfrac{1}{2} I^2 R - K(T_h - T_c)$$

where:

Q_c = net rate of heat pumped, in watts (from the cold junction)

α = Seebeck coefficients of the two materials in volts/^{o}C.

I = direct current in amperes

R = total resistance of the two materials, in ohms.

K = thermal conductance of the two materials, in watts/^{o}C.

T_h = hot junction temperature, in ^{o}Kelvin

T_c = cold junction temperature, in ^{o}Kelvin

The first term is the Peltier cooling quantity, the second is the Joule heat generated (one half goes to the hot junction and one half to the cold junction) and the last term is the heat transferred by conduction through the material.

The heat liberated at the hot junction may be expressed by the following equation:

$$Q_h = (\alpha_p - \alpha_h) T_h I + \tfrac{1}{2} I^2 R - K(T_h - T_c)$$

Here the first term is the Peltier heat released at the hot junction, the second term is one half of the Joule heat and the third is the heat conducted away from the hot junction through the material.

A relationship has been evolved between the Seebeck coefficient, thermal conductivity, and the electrical resistivity. This relationship is expressed as a factor Z or figure of merit.

$$Z = \frac{S^2}{\rho k}$$

where:

S = Seebeck coefficient in volts/^{o}C.

ρ = Electrical resistivity in ohm · cm.

k = Thermal conductivity in watts/cm.·^{o}C.

From this expression the dimension of Z is $^{o}C^{-1}$.

Important Formulas:

$$\Delta T_{max.} = \tfrac{1}{2} Z T_c^{\,2}$$

$$Q_{i\ max.} = K(\Delta T_{max.} - \Delta T)$$

$$\emptyset_{max.} = \frac{\Delta T_{max.} - \Delta T}{Z\ T_h\ T_c}$$

$$Z = \frac{(\alpha_{AB})^2}{(\sqrt{k_A \rho_A} + \sqrt{k_B \rho_B})^2}$$

where:

$\Delta T_{max.}$ = maximum temperature difference that can be produced by a couple, ^{o}C.

ΔT = operating temperature difference $(T_h - T_c)$ ^{o}C.

k = thermal conductivity - watts/cm·^{o}C.

T_h and T_c = Hot and cold junction temperatures - ^{o}Kelvin

ρ = Electrical resistivity - ohm-cm.

$Q_{i\ max.}$ = maximum cooling effect - watts.

$\emptyset_{max.}$ = maximum coefficient of performance for maximum cooling - dimensionless.

K = thermal conductance - watts/^{o}C.

Z = Figure of merit

α = Seebeck coefficient of the materials in volts/^{o}C.

APPENDIX G

List of Symbols and Explanation

LIST OF SYMBOLS USED IN THEORETICAL ANALYSIS

Symbol	Description	Units
A	area	ft.^{2}
$\bar{C}$	thermal conductance	Btu./ft.$^{2\circ}$F.hr.
h	heat transfer coefficient for flat plate	Btu./hr.ft.$^\circ$F.
I	electric current (direct current)	amperes
k	thermal conductivity	Btu./hr.ft.$^\circ$F.
L	length	ft.
m	mass flow rate	lbs./hr.
N_{Nu}	Nusselt number	dimensionless
N_{Pr}	Prandtl number	dimensionless
N_{Re}	Reynolds number	dimensionless
P_w	electric power	watts
P_{wb}	electric power	Btu./hr.
q	rate of heat transfer	Btu./hr.
q_c	cooling capacity per couple	watts
Q_c	cooling capacity of module	Btu./hr.
Q_h	heat rejected by module	Btu./hr.
R	thermal resistance	ft.^{2}hr.$^\circ$F./Btu.
t_{cf}	cold junction temperature	$^\circ$F.
t_{cc}	cold junction temperature	$^\circ$C.
t_{hf}	hot junction temperature	$^\circ$F.
t_{hc}	hot junction temperature	$^\circ$C.
t_{pc}	plate temperature on cold side	$^\circ$F.
t_{ph}	plate temperature on hot side	$^\circ$F.

t_{wa}	average water temperature	$^oF.$
Twih	inlet water temperature on hot side	$^oF.$
Twoh	outlet water temperature on hot side	$^oF.$
Twic	inlet water temperature on cold side	$^oF.$
Twoc	outlet water temperature on cold side	$^oF.$
v	velocity	ft./hr.
V	volume rate of flow	$ft.^3/sec.$
V_m	potential drop per couple	millivolts
W_h	water flow through hot junction sink	lbs./hr.
W_c	water flow through cold junction sink	lbs./hr.
x	water flow rate in heat transfer equation	lbs./hr.
y	heat transfer coefficient for flat plate related to water flow and water temperature	$Btu./hr.ft.^{2\,o}F.$
Δt_{ln}	log mean temperature difference between plate and liquid	$^oF.$
Δt_c	temperature drop of liquid due to cooling	$^oF.$
Δt_h	temperature rise of liquid	$^oF.$
Δt_{hc}	temperature difference between hot and cold junctions	$^oC.$
Δt_{cg}	temperature difference across copper plate and nylon/grease separator	$^oF.$
ν	kinematic viscosity	$ft.^2/hr.$
ρ	density	$lb._m/ft.^3$
π_1	Q_c/P_{wb}	dimensionless
π_2	$(Twih-Twic)/\Delta t_c$	dimensionless
π_3	W_c/W_h	dimensionless
π_4	$Twic/\Delta t_c$	dimensionless

APPENDIX H

Laboratory Test Data

Table 5. Basic laboratory test data.

Test No.	Millivolts				Minutes for 300 gram sample		Direct current voltage
	Twic	Δt_c	Twih	Δt_h	W_c	W_h	
			π_2 constant				
1.	1.276	0.046	1.742	0.071	2.10	1.64	0.830
	1.275	0.046	1.742	0.072	2.10	1.65	0.830
	1.275	0.046	1.741	0.073	2.11	1.64	0.830
	1.276	0.047	1.742	0.072	2.10	1.64	0.830
2.	1.280	0.046	1.742	0.048	1.99	1.11	0.825
	1.274	0.045	1.741	0.047	1.99	1.11	0.825
	1.277	0.046	1.741	0.048	1.98	1.12	0.825
	1.276	0.046	1.742	0.048	1.99	1.11	0.825
3.	1.046	0.047	1.505	0.096	2.20	2.29	0.805
	1.046	0.046	1.508	0.095	2.20	2.29	0.805
	1.047	0.046	1.506	0.096	2.21	2.28	0.805
	1.046	0.047	1.506	0.096	2.20	2.29	0.805
4.	1.048	0.046	1.505	0.058	2.10	1.38	0.795
	1.047	0.047	1.508	0.059	2.11	1.39	0.795
	1.046	0.047	1.506	0.059	2.10	1.38	0.795
	1.047	0.047	1.508	0.060	2.10	1.38	0.795
5.	0.822	0.046	1.276	0.073	2.26	1.78	0.785
	0.824	0.046	1.277	0.072	2.25	1.77	0.785
	0.823	0.046	1.276	0.073	2.25	1.77	0.785
	0.823	0.045	1.277	0.072	2.26	1.77	0.785
6.	0.822	0.046	1.276	0.049	2.15	1.18	0.780
	0.820	0.046	1.276	0.051	2.16	1.18	0.780
	0.823	0.046	1.277	0.051	2.15	1.18	0.780
	0.823	0.045	1.276	0.050	2.15	1.18	0.780
7.	0.823	0.044	1.276	0.046	2.09	1.09	0.775
	0.821	0.046	1.275	0.046	2.10	1.09	0.775
	0.821	0.044	1.277	0.047	2.10	1.09	0.775
	0.822	0.046	1.276	0.046	2.10	1.09	0.775

Table 5 (cont.).

Test No.	Millivolts				Minutes for 300 gram sample		Direct current voltage
	Twic	Δt_c	Twih	Δt_h	W_c	W_h	
			π_3 constant				
8.	0.823	0.046	1.508	0.050	2.48	1.24	0.800
	0.824	0.047	1.507	0.049	2.48	1.24	0.800
	0.823	0.046	1.507	0.049	2.48	1.24	0.800
	0.823	0.046	1.508	0.050	2.48	1.24	0.800
9.	0.824	0.046	1.336	0.048	2.32	1.17	0.775
	0.823	0.046	1.336	0.047	2.34	1.17	0.775
	0.823	0.046	1.335	0.048	2.33	1.17	0.775
	0.823	0.046	1.336	0.048	2.33	1.17	0.775
10.	0.823	0.047	1.249	0.047	2.27	1.13	0.770
	0.824	0.046	1.248	0.046	2.26	1.14	0.770
	0.822	0.046	1.249	0.046	2.27	1.13	0.770
	0.824	0.047	1.248	0.046	2.27	1.13	0.770
11.	1.048	0.047	1.856	0.053	2.50	1.25	0.845
	1.048	0.047	1.854	0.052	2.50	1.25	0.845
	1.047	0.048	1.855	0.052	2.51	1.25	0.845
	1.048	0.047	1.856	0.053	2.51	1.25	0.845
12.	1.047	0.046	1.508	0.046	2.20	1.10	0.805
	1.046	0.047	1.509	0.047	2.20	1.10	0.805
	1.048	0.046	1.507	0.046	2.21	1.10	0.805
	1.047	0.046	1.508	0.046	2.20	1.10	0.805
13.	1.046	0.047	1.710	0.048	2.42	1.21	0.820
	1.045	0.046	1.711	0.048	2.41	1.21	0.820
	1.045	0.046	1.711	0.049	2.42	1.21	0.820
	1.046	0.047	1.710	0.048	2.42	1.21	0.820
14.	1.277	0.047	2.018	0.052	2.32	1.18	0.855
	1.276	0.046	2.019	0.051	2.31	1.18	0.855
	1.277	0.046	2.018	0.052	2.32	1.18	0.855
	1.277	0.047	2.018	0.052	2.32	1.18	0.855
15.	1.277	0.046	1.838	0.048	2.20	1.10	0.830
	1.276	0.046	1.837	0.050	2.21	1.10	0.830
	1.277	0.047	1.837	0.050	2.20	1.10	0.830
	1.276	0.047	1.838	0.049	2.20	1.10	0.830

DEVELOPMENT OF A SIMPLIFIED ENGINEERING DESIGN EQUATION
FOR A THERMOELECTRIC HEATING/COOLING DEVICE

by

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AN ABSTRACT OF A MASTER'S THESIS

submitted in partial fulfillment of the

requirements for the degree

MASTER OF SCIENCE

Department of Agricultural Engineering

KANSAS STATE UNIVERSITY
Manhattan, Kansas

1966

The past decade has seen the development of new semi-conductor materials which exhibit a greater thermoelectric effect than previously was available. As a result the application of thermoelectric cooling devices may become more feasible within the next decade or so. Devices which can cool and heat without moving parts are already being manufactured.

The application of thermoelectric cooling may be feasible in agriculture for such uses as milk pasturization and/or milk cooling coupled with water or air heating or cooling.

Design of thermoelectric devices usually proceeds by selecting some cold junction temperature to be held constant. Such may not be the case with some applications as mentioned in this study. The development of simplified engineering design equations can be of help in predicting the operating characteristics of such devices, especially if the operating conditions are not constant. It was decided to investigate the application of the principles of dimensional analysis to determine the relationship of the important variables involved in a thermoelectric heating/cooling device. It was first necessary to explore theoretically the dimensionless Pi groups that could be formed in order to determine the proper combination that would yield an explicit relationship. The theoretical analysis was performed with an IBM 1620 computer.

A simple device was fabricated to handle liquid in both heat exchangers. The heat exchanger design approached that of a flat plate but with the liquid entering the chambers at right angles. An empirical equation for flat plate heat transfer was used in the theoretical analysis. Operating characteristics of the thermoelectric module were derived from the manufacturers data.

The current was held constant at 15 amperes. The theoretical analysis assumed water inlet temperatures on the cold side heat sink of 70, 80, 90 and 100°F. Flow rates were assumed between about 10 lbs/hr to about 50 lbs/hr. The temperature drop of the water being cooled was assumed to be 2°F and held constant at that value.

The data obtained from the theoretical analysis with the IBM 1620 computer was programed for multiple-regression analysis with the IBM 1410 computer. The multiple-regression analysis yielded an equation which showed the mathematical relationship of the Pi groups.

Equipment was assembled in the laboratory and analysis similar to that used with the computer program was followed. The data was then programed for multiple-regression analysis by the IBM 1410 computer.

The results of the laboratory tests were in general agreement with the relationships as predicted by the theoretical analysis.